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Essays on Female Labor Supply and Fertility Responses to Marital Dissolution

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Essays on Female Labor Supply and Fertility Responses to Marital Dissolution

by

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Dedication

This dissertation is dedicated to my grandmother whom I miss dearly.

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Essays on Female Labor Supply and Fertility Responses to Marital Dissolution

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Choices regarding labor supply and fertility by married women are generally made to maximize family welfare in harmonious marriages. However, as the prospect of marital dissolution becomes likely over time, labor supply and childbearing decisions may not be formulated in a manner that are consistent with the goal of household utility maximization, rather they are often determined to improve individual post-marital wellbeing. The current literature that addresses the effects of marital disruption on labor supply and childbearing within marriage assumes the choices made by the wife are independent of the actions undertaken by the husband and thereby ignores the possible strategic interaction between members of the household. By exploring the strategic behaviors on the part of the spouses, we find new answers to some old questions.

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Chapter 1: Literature Review and Nature of the Problem

In the basic framework of the consumer choice theory, rational agents with fixed endowments attain the goal of utility or satisfaction maximization by making tradeoffs between various goods. This approach of modeling human behavior is sensible when choices are consequential only to the wellbeing of the decisions maker; however, an individual's actions often have an impact on the welfare of the group that the individual is a part of. When the choices are constrained not only by personal but also by group preferences, it is crucial to understand the nature of interaction between members of a group in hypothesizing an individual's behavior. In this thesis, we study how people formulate plans in the context of the most intimate and important group there is: the family. In particular, the focus is on the determination of labor supply and fertility patterns for married women in response to rising likelihood of marital dissolution.

Much of the existing economic analysis either directly or indirectly deals with choices made by a family. Yet, we still do not fully understand the process through which a family, functioning as an economic unit, allocates limited resources to optimize welfare. The less-than-perfect understanding of the family is perhaps related to the relatively late emergence of the economics of family as a field. Prior to Gary Becker, family behavior was rarely examined through the lens of economics. Becker (1981) applied the neoclassical consumer theory to the analysis of family, Becker suggesting that once two people find their "match" in the marriage market and together form a household, they pool their endowments (in terms of both time and wealth) to maximize a "household utility function." The premise behind a household utility function is that husband and wife share very similar, if not identical, preferences. As a result, there are little or no distribution effects generated from various patterns in household consumption, and preferences of different household members can be aggregated into a common function.

Using the structure of the household utility function, Becker analyzed the demand for children. In his model, a family maximizes a utility function with three components: the quantity of children, denoted by N, the quality of each child, denoted by Q, and the aggregate quantity of other consumption goods, denoted by Z. The introduction of child quality as a source of satisfaction for parents distinguishes Becker's theory from previous economic studies on the demand for children. The quality of each child is assumed to be a positive function of the amount of expenditure on the child. If a couple prefers a large number of children, the average quality of children is not likely to be very high given a fixed level of income. On the other hand, if a couple cares a great deal about the quality of children, they would choose to have few kids. Hence, there is an inherent tradeoff between quantity and quality of children in Becker's analysis. Mathematically, the household utility function takes the general form of:

$$(1.1) \qquad U = U(N, Q, Z)$$

The objective of the family is therefore to maximize the utility function in (1.1) subject to the following budget constraint:

$$(1.2) \qquad P_C QN + P_Z Z = M$$

In equation (1.2), P_C, P_Z, and M stand for the constant cost of a unit of quality, the price of the aggregate consumption good, and the household income, respectively. Hence P_CQN represents the total expenditure on children and P_ZZ is the total amount spent on other consumption goods. If the utility function is maximized subject to the budget constraint with respect to the components of the utility function, three first-order conditions are derived (not shown here). From the first-order conditions, we can infer that the true cost of quantity of children depends on the quality per child since a higher average quality of children raises the necessary expenditure on each child. At the same time, the true cost of quality of children depends on the quantity of children since a larger number of kids leads to a higher cost of enhancing the quality of each child.

Suppose there is an exogenous increase in the family income that induces a higher demand for both quantity and quality of children[1]. As mentioned before, an increase in N adds to the relevant cost for quality and in turn lowers the demand for quality. Also, an increase in Q raises the shadow price of quantity and leads to a lower demand for quantity. If the income elasticity of demand for quality is sufficiently greater than the income elasticity of demand for quantity, the negative substitution effect for quantity produced by a higher shadow price may dominate the positive income effect for quantity. The net effects of an increase in income, in this case, are higher quality and lower quantity of children. The quantity-quality tradeoff argument offered by Becker provides a critical explanation for the negative relationship between fertility and wealth observed in the last century or so.

[1] Both quantity and quality of children are assumed to be normal goods.

The assumption that all members of the same household have the same preferences is not entirely realistic. Becker (1981), however, argued that as long as one member of the family is altruistic, the rest of the household would behave as if they were maximizing the altruist's utility.

Becker defined an altruist as someone who cares about the welfare of others in the household while a selfish person is characterized as someone who has no regard for the wellbeing of others. Suppose that person A and person S are the only two members in the household with A being altruistic and S being selfish. A's utility function takes on the form of

$$(1.3) \quad U_A = U_A(Z_A, F(U_A)) \qquad \text{with } ?U_A / ?U_S > 0$$

where Z_A is the consumption of a composite good by A and F is a positive function of U_S with U_S being S's utility function. By definition, S's utility depends only on it's own private consumption, Z_S. Since S's welfare enters A's utility function, A spends part of the income on S. Effectively, A maximizes equation (1.3) subject to the following family budget constraint:

$$(1.4) \quad Z_A + Z_S = M_A + M_S$$

where M_A and M_S represent the income of A and S and the price of Z is set equal to unity. As long as S's utility is a normal good to A, A would raise the level of spending on S as the household income increases. Knowing that the household income constraint dictates the amount of transfers from the benefactor to the beneficiary, S is likely to take actions that maximize the joint family income. If A is sufficiently altruistic and has a relatively high level of income, S may even sacrifice his or her own consumption initially in order to boost the household resources. The insight that the selfish family member strives to maximize the income relevant to the altruist's utility maximization problem, even though the selfish person does not gain satisfaction from the altruist's happiness, is known as the Rotten Kid Theorem.

To see how the Rotten Kid Theorem works, we consider a simple example mentioned by Becker using the two-person household introduced previously. Suppose S likes to eat using their fingers, but the act of eating with fingers by S annoys A. S is aware of the consequences of eating with fingers and decides not to do it. Becker's explanation for S's behavior is as follows. A's utility would be lowered if S eats with fingers. As A's utility is decreased by S's action, A is likely to reallocate spending in a way that raises A's own consumption and lowers the transfer amount to S. If the initial decrease in A's utility from S eating with fingers is larger than the initial increase in S's utility, the family income essentially goes down. Since there is a reduction in the family income and S's utility is a normal good, A would reduce the transfer by a larger amount than the monetary-equivalent increase in S's utility from eating with fingers. Being fully aware of the potential of being worse off, S would not choose to eat with fingers.

The rationale behind the Rotten Kid Theorem is both elegant and compelling; however, it is not generally correct, as shown by Theodore Bergstrom (1989). It turns out that the validity of the Rotten Kid Theorem critically depends on the implicit assumption of transferable utility. In the tale of eating with one's fingers, Becker presumed that S's action only creates a pure income effect, i.e., the "utility possibility frontier" only shifts inward in a parallel fashion as S eats with fingers. However, Bergstrom proved that S's action causes a substitution effect as well as an income effect unless transferable utility is assumed. Transferable utility essentially means that the sum of utilities within a group can be distributed in every possible way without the sum being altered. Without the assumption of transferable utility, S can manipulate the slope of the utility possibility frontier in a manner that does not maximize the "family income" but is somehow favorable to S.

In the case when the singular-utility-function household cannot be justified, we need to recognize that choices in a family are made through a process of intricate interaction among the members and that the outcomes yielded in this manner may differ significantly from those generated from the "unitary" model. Hence, the pertinent question is how should we formulate the decision-making process within a household that accounts for distinct preferences of various members. Manser and Brown (1980)[2] and McElroy and Horney (1981) emphasized the importance of bargaining within a household and adopted the Nash cooperative bargaining approach as the appropriate model. In their models, a household is composed of a husband and a wife and each has a unique utility function as follows.

(1.5) $U_i = U_i(Z_i, R_i, G)$ for $i = \{husband, wife\}$

where Z_i is i's private consumption good, R_i is i's leisure, and G is a shared (public) consumption good. Since leisure is a component of the utility function, each individual

[2] According to Manser and Brown, Becker's Rotten Kid story is a special case of bargaining where the implicit rule is utility maximization of the altruist in the household.

faces a time constraint as well as a budget constraint. Equation (1.6) and (1.7) specify the time constraint and the budget constraint, respectively.

(1.6) $R_i + L_i = T$

where L_i is the labor supply by i and T is the time endowment.

(1.7) $P_Z Z_i + P_G G = W_i L_i + M_i$

where P_z and P_G are the price of good A and good G, respectively and M_i is i's money endowment.

The utility function itself is assumed to be independent of marital status (i.e., preferences remain constant); however, the level of utility is related to whether the individual is currently part of a household or not. In the single state, $U_i = V_i$ where V_i can be interpreted as i's threat point in negotiation. The threat points V_h and V_w are crucial in solving for the two-person bargaining game. According to John Nash (1953)[3], the solution is the combination of utilities, U_h* and U_w*, that maximize the product of $(U_h - V_h)$ and $(U_w - V_w)$. McElroy and Horney showed that unless the threat points are invariant to changes in commodity prices and incomes, household demand functions cannot be reconciled with the single-agent model.

The unitary household decision-making model is not only problematic on the theoretical ground; it has also been challenged empirically. In an interesting study using a socioeconomic survey of Thailand, T. Paul Shultz was able to reject the income-pooling assumption under the unitary model. He found that, holding the total level of household income constant, women with more nonearned income are more likely to have greater consumption of leisure (or time in nonmarket activities) and more children[4]. It is worth noting that although Schultz's study invalidated the neoclassical common preference model, it did not fully accept the Nash bargaining model due to the lack of the exclusion restrictions needed to correct for sample selection bias.

The notion that the threat points in bargaining are the utilities attainable outside marriage in the Manser-Brown/McElroy-Horney model is not universally accepted. Lundberg and Pollak (1993) argued that in the presence of transaction costs, both financial and mental, divorce is not always a feasible option when bargaining breaks down. Instead, the outcome is perhaps "a noncooperative equilibrium defined in terms of traditional gender roles and gender role expectations." The specification of threat points in the Lundberg and Pollak framework produces drastically different implications from that in the divorce threat bargaining model regarding the distribution effects of government child allowances. Allocation of gains from marriage in the Nash bargaining depends critically on each spouse's threat point: whoever has the higher threat point extracts more surpluses from bargaining. Government childcare allowances do not affect welfare of individuals once marriage dissolves. Hence, if the utility in the state of divorce is indeed the threat point in bargaining, it is irrelevant distribution-wise whether childcare allowances are paid to mothers or fathers. On the other hand, if the threat point

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[3] In the paper, Nash presented a set of axioms that the game must satisfy.

[4] The conclusion with regard to the childbearing behavior is more qualified. Shultz only established the positive correlation between transfer income to the women and their fertility choice, but could not find a statistically significant relationship between their property nonearned income and fertility. Thus, it is very possible that the causation between nonearned income and number of kids is the reverse, that is, women with more kids tend to receive greater amount of transfers from their relatives and other sources.

involves some noncooperative equilibrium within marriage, the person who receives the allowance is likely to have a higher threat point as a result of the transfer. As the threat point for the individual is enhanced, she or he enjoys a higher share of household resources.

Lundberg and Pollak also cautioned that perhaps only the existing marriages at the time of policy change are affected by a government program that alters allocation of resources within households. The redistributive effects of such program may be either completely or partially undone in marriages that are formed after the policy is implemented. Suppose that child allowance payments are directly granted to women. If binding pre-marital contracts can be negotiated in the marriage market to specify the distribution of gains from marriage, the government policy will lower the share of resources entitled to wives in future marriages by exactly the amount of the allowance. On the other hand, if there are no binding prenuptial agreements in place, the government policy will increase the number of women and decrease the number of men who want to get married. As a result, the equilibrium number of marriages may either increase or decrease in the subsequent marriage market, and, in turn, the distribution of marital gains within households may change as well.

The cooperative bargaining approach of modeling decision-making within households is appropriate when marriages are in harmony; however, as tensions begin to mount within marriage, the element of cooperation in the everyday functioning of a family may disintegrate. Hence, the Nash cooperative bargaining game is probably not suitable in analyzing household behavior when marriage becomes unstable. As marital separation looming on the horizon, strategic interaction is likely to develop between members of a family. Therefore, a non-cooperative game is needed to study choices made by individuals in a fractured household. Using a two-person intertemporal Cournot game, this thesis addresses a special type of problem that tends to arise in unstable marriages: the issue of moral hazard.

H. Elizabeth Peters (1986) speculated that the problems associated with moral hazard that are often cited in the labor market literature exist in the marital relationship as well. In the context of marriage, moral hazard refers to the phenomenon when less-than-optimal level of investment in marriage-specific capital is made due to the possibility of marital dissolution. Historically, men have higher wage rates than women[5]. Hence, in order to take advantage of specialization within households, husbands tend to work in the labor market while wives are likely to work at home[6]. The opportunity costs of engaging in marriage-related activities such as housework and childcare[7] are wages forgone both in the present and in the future (in the form of labor market experience). Women, who specialize in home production, bear the full costs in household upkeep and child-rearing while enjoying only partial benefits. When marriages are harmonious, women perform the tasks in which they have comparative advantages to maximize the household welfare. On the other hand, if marriages are falling apart, women are less willing to fulfill their assigned "duties" since they need to insure against the state of divorce by working more in the labor market.

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[5] The discussion about why men have greater earning powers than women is extensive and outside the scope of this paper.

[6] In the bargaining framework, we can interpret division of labor within households as the outcome of efficient negotiation.

[7] Raising children and maintaining an orderly house are obviously not mutually exclusive activities. One can certainly argue that it is very difficult if not impossible to keep a house in order if there are children around.

Two types of female responses to the rising likelihood of marital disruption are examined in this thesis: labor supply and fertility. Both the labor supply and fertility choices associated with the problem of moral hazard within marriage have previously been studied. Johnson and Skinner (1986) showed that increasing probabilities of divorce lead women to raise both labor force participation and work hours and the changes in behavior are particularly significant among those who have limited experiences in the labor market. Lillard and Waite (1993) found that the hazard of marital disruption has a strong negative effect on the hazard of marital childbearing, and this effect is especially pronounced for women who have had at least one child. Although illuminating, both papers are empirical in nature and are not based on explicit models. Here, a theoretical framework is developed to explore the link between labor supply, childbearing, and marital stability. The implications derived from the model are then used to guide the empirical analysis of the issues of interest.

The general set-up of the model is sketched below to further our understanding of the moral hazard problem that can potentially arise in the marital relationship. We assume that two individuals, i and j, decide to enter marriage in period 0 upon observing the characteristics of each other in the marriage market. The reasons why they choose the other person instead of someone else as a partner are not explored. As a matter of fact, the marriage market is assumed to be completely exogenous throughout the analysis. Consequently, the choices made within marriage are independent of the potential payoffs one can receive in the marriage market, i.e., no remarriage is allowed in the model. Once married, the couple plays a non-cooperative two-person game repeatedly. The game ends when one party chooses to divorce, or when both spouses perish simultaneously in period

T. Following the Beckerian approach of modeling the demand for children, each individual k's objective in a given period t is stated as below:

(1.8) $\max U_t^k = U_t^k(N_t, Q_t, Z_t, ?_t) \qquad \forall k \in \{i, j\}$

N_t denotes the cumulative quantity of children dating back to period 0. In other words, $N_t = \sum_{t=0}^{T} n_t$ (or $N_t = N_{t-1} + n_t$), where n_t is the number of children added to the household in period t. n_t is jointly determined by the husband and the wife, hence $n_t = f(n_t^i, n_t^j)$ with $f' > 0$ (with respect to both n_t^i and n_t^j). Q_t is the quality per child in period t. The quality of children can only be manufactured using parents' time. Unlike N_t, Q_t is non-cumulative; it represents solely the amount of time that parents invest in their children in that period[8]. Both the quantity and the quality of children are pure public goods since the consumption of which is neither rival nor excludable. If individual i devotes R_t^i units of time nurturing kids, both i and j would enjoy a level of child quality equal to $g^i R_t^i / N_t$ in period t where $?^i$ is an efficiency parameter indicating how well i transforms the input of time into the output of child quality. Notice that "home production" in the model involves only nourishment of children and nothing else. All other goods consumed by the household are lumped under the category of Z_t that is purchased with wages earned in the labor market. The total quantity of Z_t purchased by the household is the sum of Z_t^i

[8] If we regard the quality of children as a form of investment, Q_t is said to have a depreciation rate of 100% in each period.

and Z_t^j where Z_t^i and Z_t^j are bought by the income earned by i and j respectively. The utilities generated from the consumption of Z_t, however, take on the nonlinear form of $Z_t = g(Z_t^i, Z_t^j)$. We assume that Z_t^i and Z_t^j are enjoyed by i at different intensities, i.e., Z_t^i and Z_t^j enter i's utility function in the form of ${Z_t^i}^{a_i}$ and ${Z_t^j}^{b_i}$ with $a^i > b^i$. Z_t hence is a "quasi-public" good in the sense that both i and j derive utilities from the proportion of Z_t purchased with i's labor income, but i enjoys Z_t more than j does[9]. One type of household consumption that can be classified as a quasi-public good is the expenditure on housing. All members of the household gain satisfaction from the residential unit. However, the person who bears the cost of housing is likely to get the largest bedroom and generally has more control over the usage of the house. Finally, $?_t$ is a parameter that represents the level of compatibility between the couple. $?_t$ is the only stochastic element in the model, it is equal to $?_{t-1} + e_t$ where e_t is a periodical random shock with the standard normal distribution.

In maximizing utilities, each individual faces the following constraints in each period:

(1.9) $$Z_t^k = w_t^k L_t^k$$

[9] Becker's altruist argument provides an alternative interpretation to Z_t being a quasi-public good. Under the altruist interpretation, we can assume that each person only consumes the proportion of Z_t that is purchased by her/his own labor income. Although Z_t is a private good that is consumed solely by the person who directly acquires it, the consumption of Z_t by j can still enter person i's utility function (and vice-versa) since i cares about the welfare of j. Also, in general, each individual weighs their own utilities more than the spouse's utilities, hence $a^i > b^i$ and $a^j > b^j$. The altruist interpretation, however, is not entirely appealing since it implies that other than children, a full-time housewife (or househusband for that matter) does not have any direct private consumption and that the utilities from Z_t are solely derived from the husband's consumption.

(1.10) $N_t Q_t = g^i R_t^i + g^j R_t^j$

(1.11) $L_t^i + R_t^i = H$

(1.9) is the individual budget constraint which states that the consumption good, Z_t^k, can only be purchased by one's own labor income. w_t^k is k's wage rate in period t and it equals to $w_0^k + w(L_{t-1}^k)$ with $w' > 0$. This means that the current wage rate is a function of labor supply from the previous period and the individual's "innate" ability[10]. Also, note that the labor income earned has to be exhausted in the same period, hence savings are not permitted in the model. The technological constraint for home production is expressed by (1.10). R_t^i and R_t^j denote the number of hours that i and j spend on nurturing children respectively. As mentioned before, ?^{i} and ?^{j} are efficiency parameters in home production. Unlike the wage rates, they are assumed to be invariant to the relevant experiences gained in the past; i.e., they are fixed in each time period. (1.11) represents the time endowment constraint. It places a limit (H) on the total amount of time available to an individual to work in the market and/or at home in each period.

Individual i's complete intertemporal optimization problem within marriage is expressed as follows:

(1.12) max $$E_0\{\sum_{t=0}^{T} b^t U_t^i(N_t, Q_t, Z_t, q_t)\}$$

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[10] If we assume that both partners had completed their schooling before marriage, w_0^k can be interpreted as individual i's educational level.

subject to $N_t = N_{t-1} + f(n_t^i, n_t^j)$

$$Q_t = (g^{\,i} R_t^i + g^{\,j} R_t^j)/N_t$$

$$Z_t = g(Z_t^i, Z_t^j) = g[(w_t^i L_t^i), (w_t^j L_t^j)]$$

$$q_t = q_{t-1} + e_t$$

The stochastic variable, ?$_t$, determines whether the marriage dissolves in each period. Once observing ?$_0$, the compatibility factor in the initial period, the couple gets married while taking the expected lifetime utility generated from the union into consideration. If ?$_t$ is deterministic (coupled with the assumption of no remarriage), the couple would choose to stay married until the terminal period T[11]. However, since ?$_t$ changes unexpectedly over time, one or both individuals may want a divorce if the level of compatibility becomes sufficiently low. Once divorced, each individual faces the intertemporal problem below:

(1.13) max $E_0\{\sum_{t=D}^{T} b^t U_t^k(N_t, Q_t^k, Z_t^k)\}$ where D is the date of divorce

[11] In a permanent income hypothesis framework without random shocks, the optimal rule is to have a constant level of consumption in every period. Similarly, the optimal decision rule regarding marriage is unvarying over time if ?$_t$ is deterministic. Since the couple made the decision to get married in the first place, remaining in the marriage must always be the optimal choice. Suppose someone dissolves the marriage in a subsequent period, it follows that the couple would not choose to be together initially. The additional assumption of no remarriage is also crucial to the time-invariant optimal rule of staying married since without the assumption, it is possible for more suitable mates to appear in later periods.

subject to $N_t = N_{t-1}$

$$Q_t^k = g^k R_t^k / N_t$$

$$Z_t^k = w_t^k L_t^k$$

As marriage disintegrates, both the quality of children and the composite consumption good become private instead of public in nature. It is easy to see how a quasi-public good like housing loses its public property in the case of marital disruption since divorcees generally do not share the same dwelling. But, it is more difficult to imagine how a pure public good such as child quality can turn private when the couple separates. It is perhaps a bit of a stretch to deny the existence of external effects from investment in children in the state of marital separation. However, there may be a loss in the economies of scale associated with childcare when the family is fragmented; the transformation of the child quality from a public good to a private commodity is meant to capture that loss.

The absence of positive external effects in consumption in the state of divorce leads to, essentially, a decline in real income for both parties. The person who previously specialized in home production within the marriage becomes susceptible to negative consequences of a low wage (unless the individual has a high w_0). According to Peters (and confirmed empirically to certain extents by Johnson-Skinner and Lillard-Waite), the problem of moral hazard arises when the homemaker guards against the adverse effects of a divorce by working more in the labor market as well as giving fewer births than she otherwise would. Suppose person i is the one who concentrates on home production. In the current model, moral hazard means that L_t^i (labor supply) would go up and n_t^i (number of newborn kids) would go down as responses to a decreasing ? (compatibility level), that is $\frac{\partial L_t^k}{\partial E(q_{t+1})} < 0$ and $\frac{\partial n_t^k}{\partial E(q_{t+1})} > 0$. However, as we will find out later, the optimal responses of housewives to marital instability are not always those suggested by Peters. Indeed, the issue of "moral hazard" is more convoluted than previously thought.

In Chapter 2, we assume that the number of kids is constant for the duration of the marriage. We use a general term, C_t, which denotes the aggregate pure public good in the household in place of the gross quality of all children, N_tQ_t. Once each agent's problem is reduced to an intertemporal tradeoff between working in the market and working at home, we can focus on the relationship between marital instability and labor supply. The restriction of fixed number of kids in the household is relaxed in Chapter 3 to allow the examination of marital disruption and fertility behavior. Finally, in Chapter 4, both the theoretical and empirical findings are summarized and the directions for future research are discussed.

Chapter 2: A New Look at Labor Supply and Marital Separation: The Household Production Approach

2.1 Introduction

In this chapter, we focus on how the expectation of marital separation may alter the labor market behavior of women. The issue of female labor supply response to divorce had been examined previously by several studies. In the most comprehensive analysis to date, Johnson and Skinner (1986) demonstrated empirically that women who experience rising probabilities of divorce are likely to increase labor supply. Johnson and Skinner, however, did not fully consider the effect of reallocation of time on the value of marriage. In light of the limitations of the current literature, a two-person dynamic game is constructed to explore the labor supply response to marital disruption. The predictions derived from the intertemporal marriage game are significantly different from those of the prevailing view regarding divorce and the time-allocation choice made by women.

The remainder of the paper is organized as follows. Section 2.2 presents the static version of the game. The dynamic version of the model is outlined in Section 2.3. Section 2.4 addresses the effect of changes in labor supply on the stability of marriage. Section 2.5 focuses on how labor supply decisions are influenced by the prospect of marital dissolution. Section 2.6 explains how the theoretical predictions obtained from Section 2.4 and 2.5 can be tested empirically. In Section 2.7, the National Longitudinal Survey of Youth 1979 (NLSY79), the data set used to verity the validity of the theory is described and the preliminary results are presented. The econometric techniques utilized to estimate the model are presented in Section 2.8. The results from the estimation are discussed in Section 2.9. Finally, Section 2.10 concludes.

2.2 The Static Model

In order to analyze how the members of a family would modify labor supply behavior over time; we need an intertemporal framework in which the choices made by the husband and the wife are interdependent. However, before we proceed to formulate the full dynamic model, it is helpful to examine the problem faced by the couple in the one-period game.

According to Yoram Weiss (1997), there are four economic factors that contribute to the formation of a family by two individuals other than the production and rearing of children. These factors are (1) sharing of collective goods, (2) division of labor to exploit comparative advantage, (3) extending credit and coordination of investment activities, and (4) risk pooling. In this chapter, we assume the exogeneity of the fertility decision. Rather, we attempt to incorporate the aspects of family that are related to (1) and (2) as well as an important non-economic motive for marriage, namely, the affection the companions have for each other.

Suppose a man and a woman form a family, each member's static decision is to:

$$
\begin{aligned}
(2.1)\qquad & \max \quad U^i(m) = a^i \ln Z^i + b^i \ln Z^j + \ln C + q \\
& \text{subject to} \quad Z^i = w^i L^i \\
& \qquad\qquad\quad Z^j = w^j L^j \\
& \qquad\qquad\quad C = g^i R^i + g^j R^j
\end{aligned}
$$

$$L^i + R^i = H$$

$$L^j + R^j = H$$

where $U^i(m)$ = person i' s utility in the state of marriage

Z^k = the market good purchased by person k (where k $\in$ {i, j})

C = the household good

L^k = the number of hours worked in the market by person k

R^k = the time devoted to the production of the household good by person k

w^k = the wage rate of person k

g^k = person k's efficiency parameter in producing the household good

H = the total amount of time available in each period

a^i = person i's intensity parameter in consuming the market good purchased by i

b^i = person i's intensity parameter in consuming the market good purchased by j

θ = the compatibility parameter between the couple

Each person derives utilities from two types of commodity: the market good (Z) and the household good (C). The market good is obtained from the incomes earned in the labor market. As mentioned in Chapter 1, Z is a quasi public good that is enjoyed by both members of the household. However, each person gains more satisfaction from each unit of Z that is purchased by his or her own earnings, that is, $a^k > b^k$ where $k \in$ {i, j}. The household good is a pure public good that can only be manufactured with each individual's time. g^i and g^j, the technology parameters, determine the rate at which the input of time is transformed into the output of the household good. Under this formulation, each person's problem is to allocate the fixed amount of time between working in the labor market and producing at home given the wage rates and the efficiency parameters. Hence, a more straightforward way to represent the individual's single-period optimization problem within marriage is:

(2.2) $$\max_{L^i} U^i(m) = a^i \ln w^i + a^i \ln L^i + b^i \ln w^j + b^i \ln L^j + \ln[g^i(H - L^i) + g^j(H - L^j)] + q$$

In the case when the person is divorced, his/her optimization problem is:

(2.3) $$\max_{L^j} U^i(d) = \ln w^i + \ln L^i + \ln g^i + \ln(H - L^i)$$ [12]

The first-order conditions derived from the within-marriage optimization problem for person i and j, respectively, are:

(2.4) $$\frac{a^i}{L^i} - \frac{g^i}{g^i(H - L^i) + g^j(H - L^j)} = 0$$

(2.5) $$\frac{a^j}{L^j} - \frac{g^j}{g^i(H - L^i) + g^j(H - L^j)} = 0$$

Combining equation (2.4) and (2.5), we find the following relationship between L^i and L^j:

[12] Notice that the division of labor does not exist in the state of divorce. As a result, increasing returns in the production of the household good can no longer be exploited.

(2.6) $$L^i = \frac{a^i}{a^j}\frac{g^j}{g^i}L^j$$

Equation (2.6) captures the essence of specialization in households that was discussed by Becker (1981). Becker suggested that in order to exploit comparative advantage, all household members (possibly except for one) should specialize in either the market or household sector. According to equation (2.6), if person i is more efficient at home production than person j ($\gamma^i > \gamma^j$), then person i would work more at home and less in the market than person j. On the other hand, if person i is less competent at home production than person j ($\gamma^j > \gamma^i$), then person i should spend less time at home production and more time in the labor market. Complete specialization in the household, as hypothesized by Becker, is however not implied by equation (2.6) because of the log-linear utility function assumed which restricts Z^i and Z^j to be strictly greater than zero.

Becker's account of the division-of-labor story, as we have shown, is static. It does not tell us how each household member's decision rule regarding time allocation is modified over the duration of the marriage. When two people decide to form a family, expectation about their union is formed. However, if the marriage turns out to be less than desirable, they may wish to separate. As the prospect of divorce becomes likely, women who previously chose to "specialize" in home production may no longer be willing to do so. They may begin to participate in the labor market to insure against the negative consequences associated with marital dissolution. The intertemporal version of the model will be introduced in the next section to enable the analysis of fluctuations in labor supply over time.

2.3 The Dynamic Model

In the dynamic model, we assume that a couple enters marriage in period 0 upon observing the compatibility parameter, q_0. Without a divorce, the marriage lasts until period T, the terminal period, in with both the wife and husband are assumed to pass away. After the initial period 0, the compatibility parameter in each period, q_t, equals to $q_{t-1} + e_t$, where e_t is a random shock to compatibility and $e_t \sim N(0, \sigma^2)$. In such a setting, each person's problem is to

$$\text{(2.7)} \quad \max \quad E_0\{\sum_{t=1}^{T} b^t U_t^i (L_t^i, L_t^j)\}$$

$$\text{subject to} \quad q_t = q_{t-1} + e_t$$

$$w_t^i = w_o^i + w^i (L_{t-1}^i)$$

$$w_t^j = w_o^j + w^j (L_{t-1}^j)$$

$$w_t^{k'} > 0, w_t^{k''} < 0, w_0^k > 0 \quad \forall k \in \{i, j\}$$

Under this formulation, the control variable for each individual is the level of labor supply provided and the state variables are marital status, the wage rate, and the compatibility parameter. Following the decision to start a family, the couple simultaneously splits time between working in the market and producing at home. After they make the time allocation choices, the compatibility factor in period one, q_1, is revealed. Once realizing the new compatibility parameter, each person then chooses whether or not to dissolve marriage. To sum up, there are three events in each time period and they occur in the following sequence: (1) the realization of q_t, (2) the decision about marital dissolution, and (3) the determination of labor supply. Once the marriage is terminated, both parties will remain single eternally. In other words, we do not allow remarriage in the model[13]. Thus, if the marriage is terminated, each person's intertemporal problem becomes one of deterministic rather than stochastic in nature.

At any period t, the person's value function is defined as follows (we denote the set of state variables in period t as S_t):

$$(2.8)\qquad V_t^i(S_t) = \max_{L_t^i} U_t^i(L_t^i, S_t) + bE[V_{t+1}^i(S_{t+1}) \mid L_t^i, S_t]$$

$$= \max_{L_t^i} a^i \ln w_t^i + a^i \ln L_t^i + b^i \ln w_t^j + b^i \ln L_t^j + \ln[g_t^i(H - L_t^i) + g_t^j(H - L_t^j)]$$

$$+ q_t^i + b\{P(L_t^i)V_{t+1}^i(m) + [1 - P(L_t^i)]V_{t+1}^i(d)\}$$

where b = the discount rate

$E[\cdot]$ = the expectation operator

P = the probability of staying married[14]

$V^i_{t+1}(m)$ = the value function at t+1 if the person is married

$V^i_{t+1}(d)$ = the value function at t+1 if the person is divorced

[13] In the conclusion, we will discuss the ramification of relaxing this restriction.

[14] We will refer to the probability of staying married in each period as the probability of marriage hereafter.

The particular comparative statics we are interested in is $\partial L^i_t/\partial E(q_{t+1})$, that is, how one's labor supply is affected by changes in the expected compatibility between the couple. Although individuals may very well respond to marital instability by altering labor market behavior, it is also possible that fluctuations in time allocation can increase the risk of divorce. In other words, the direction of causation between labor supply and marital dissolution is ambiguous. Instead of making a priori assumptions regarding the nature of labor supply and marital choices, we will explicitly sort out how they influence each other. Hence, before proceeding to solve for $\partial L^i_t/\partial E(q_{t+1})$, we will find the effect of changes in labor supply on the decision to dissolve marriage in the next section.

2.4 Effect of Changes in Labor Supply on the Probability of Divorce

As mentioned in Chapter 1, without q_t, the only stochastic element in the model[15], the couple would always keep the marriage intact and the amount of time devoted to the labor market would be allocated in a fashion as to smooth the lifetime consumption. Thus, the realized value of the compatibility determines the fate of the marriage in each period. If q_t were sufficiently high (i.e., above some critical value $q_t^i *$), then person i would choose to stay in the marriage; otherwise, i would choose to divorce. In this section, we demonstrate how the critical compatibility value, $q_t^i *$, is affected by the levels of labor supply in the previous period, L^i_{t-1} and L^j_{t-1}. However,

[15]In reality, marital separation can be attributed to many different types of random shocks, such as an unexpected decline in earning power due to an illness or injury. Such unforeseen incidents, however, do not exist in our model.

before we can do that, we need to accomplish the following: (1) establish the uniqueness of q_t^i* by proving the concavity of the value function $V^i_t(q_t)$, and (2) characterize q_t^i*.

***Claim 1*:** $V^i_t(q_t)$ is a concave function of q_t.

<u>*Proof:*</u> Proof by induction on t.

The single-period utility function, $U^i(q)$, from equation (2.1) is clearly concave in q. The value function in the last period, $V^i_T(q)$, which equals to $U^i(q)$, is then also concave. We assume that $V^i_t(q)$ is concave in q for t = T, T-1,…,t+1. In order to prove that V^i_t is concave in q, we must show that $V^i_t[?q_1 + (1- ?)q_2] \geq ?V^i_t(q_1) + (1-?)\, V^i_t(q_2)$.

Let $V^i_t(q_1) = U^i_t(q_1) + EV^i_{t+1}(q \mid q_1)$ and $V^i_t(q_2) = U^i_t(q_2) + EV^i_{t+1}(q \mid q_2)$. Since $q_t = q_{t-1} + e_t$ and $e_t \sim N(0, \sigma^2)$, it must be the case that $E(q_{t+1} \mid q_t) = q_t$. Hence, we have

(2.9) $V^i_t(q_1) = U^i_t(q_1) + EV^i_{t+1}(q_1)$

(2.10) $V^i_t(q_2) = U^i_t(q_2) + EV^i_{t+1}(q_2)$

From the definition of Bellman's equation, we get the following:

(2.11) $V^i_t[?q_1 + (1- ?)q_2] \geq U^i_t[?q_1 + (1- ?)q_2] + EV^i_{t+1}[?q_1 + (1- ?)q_2]$

From the assumption we made about the concavities of U^i_t and V^i_{t+1}, equation (2.12) follows below:

(2.12) $V^i_t[?q_1 + (1- ?)q_2]$ ³ $?U^i_t(q_1) + (1- ?)U^i_t(q_2) + ?EV^i_{t+1}(q_1) + (1- ?)EV^i_{t+1}(q_2)$

If we rearrange equation (2.12), we can see that:

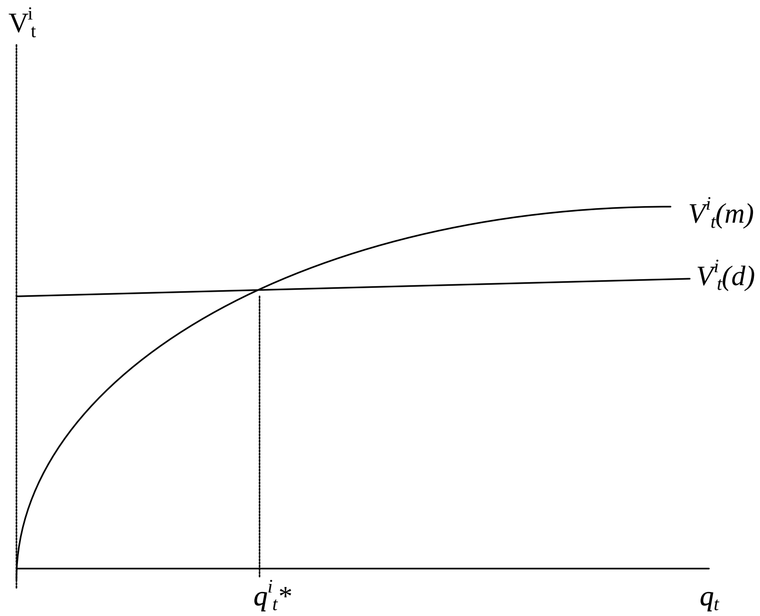
(2.13) $V^i_t[?q^i_1 + (1- ?)q^i_2]$ ³ $?V^i_t(q^i_1) + (1- ?)V^i_t(q^i_2)$

End of proof.

Once the concavity of the value function $V^i_t(q)$ is established, we plot $V^i_t(m)$, the value function in the state of marriage in period t, against $V^i_t(d)$, the value function in the state of divorce in period t, on the same graph in Figure 2.1 below. Since $V^i_t(d)$ is independent of q, it is simply represented by a horizontal line in the graph. From Figure 2.1 (on the next page), we can clearly see that $V^i_t(m)$ and $V^i_t(d)$ intersect at the point where the horizontal coordinate is q_t^i*. Hence, we find a unique compatibility value such that individual i would choose to stay married if the realized q_t exceeds that value and divorce if the realized q_t is below that value. We summarize the optimal policy regarding the divorce decision in *Claim 2*.

***Claim 2*:** If $q_t < q_t^i*$, where q_t^i* is the critical compatibility value at period t, person i would choose to get a divorce. Otherwise, person i would remain in the marriage.

<u>Figure 2.1:</u> Optimal Marital Policy



Once the compatibility parameter q_t is observed at the beginning of the period, each spouse faces the marital dissolution decision. The person would choose to remain married if the expected lifetime utility from staying in the marriage, $V^i_t(m)$, exceeds the expected lifetime utility from initiating a divorce, $V^i_t(d)$. On the other hand, the person would want to separate if $V^i_t(d)$ is greater than $V^i_t(m)$. Hence, another way to represent the value function at period t is:

(2.14) $V^i_t(q_t) = max\{U^i_t(d) + bV^i_{t+1}(d), U^i_t(m) + bV^i_{t+1}(q_{t+1} \mid q_t)\}$

We now proceed to characterize person i's critical compatibility value q_t^i*. From equation (2.14), we know that the following identity holds true at person i's critical compatibility value q_t^i*:

(2.15) $U^i_t(d) + bV^i_{t+1}(d) = U^i_t(m) + bV^i_{t+1}(q_{t+1} \mid q_t^i*)$

Substituting expressions from equation (2.2) and (2.3) into equation (2.15), we have the following:

(2.16) $$\ln w_t^i + \ln L_t^i(d) + \ln g_t^i + \ln(H - L_t^i(d)) + bV_{t+1}^i(d)$$
$$= a^i \ln w_t^i + a^i \ln L_t^i(m) + b^i \ln w_t^j + b^i \ln L_t^j(m)$$
$$+ \ln[g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))] + q_t^i* + bV_{t+1}^i(q_{t+1}^i | q_t^i*)$$

where $L^k_t(d)$ and $L^k_t(m)$ are the number of hours individual k (k $\in$ {i, j}) works in the labor market in the divorce and marriage states respectively.

From equation (2.16), we can characterize the critical compatibility level q_t^i* as below:

(2.17) $$q_t^i* = \ln w_t^i + \ln L_t^i(d) + \ln g_t^i + \ln(H - L_t^i(d)) + bV_{t+1}^i(d)$$
$$- a^i \ln w_t^i - a^i \ln L_t^i(m) - b^i \ln w_t^j - b^i \ln L_t^j(m)$$
$$- \ln[g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))] - bV_{t+1}^i(q_{t+1} | q_t^i*)$$

Now that we have determined the exact value of q_t^i*, we can perform various comparative statics to find out how the stability of marriage is affected by changes in each household member's labor supply. Here, we only state the results of comparative statics. The derivation can be found in Appendix 2.1.

Result 2.1: $$\frac{\partial q_t^k*}{\partial L_{t-1}^k} > 0 \ , \ \frac{\partial^2 q_t^k*}{\partial L_{t-1}^{k\ 2}} < 0 \ , \ \frac{\partial^2 q_t^k*}{\partial L_{t-1}^k \partial L_{t-1}^l} = 0 \ , \ \frac{\partial q_t^k*}{\partial L_{t-1}^l} < 0 \ , \ \frac{\partial^2 q_t^k*}{\partial L_{t-1}^{l\ 2}} > 0$$

$$\forall k, l \in \{i, j\}$$

q_t^i* represents the required level of compatibility that allows individual i to stay in the marriage. Thus, individual i is less likely to remain married the higher the level of q_t^i*. In other words, the probability of staying in the marriage decreases as q_t^i* goes up. If we denote the probability of staying married in period t (from the perspective of individual i) as P_t^i, we can rewrite *Result 2.1* as:

Result 2.2: $$\frac{\partial P_t^k}{\partial L_{t-1}^k} < 0 \ , \ \frac{\partial^2 P_t^k}{\partial L_{t-1}^{k\ 2}} > 0 \ , \ \frac{\partial^2 P_t^k}{\partial L_{t-1}^k \partial L_{t-1}^l} = 0 \ , \ \frac{\partial P_t^k}{\partial L_{t-1}^l} > 0 \ , \ \frac{\partial^2 P_t^k}{\partial L_{t-1}^{l\ 2}} < 0$$

$$\forall k, l \in \{i, j\}$$

Result 2.2 states that given everything else held constant, each individual would be less likely to keep the marriage intact the greater the level of his/her labor supply

($\frac{\partial P_t^k}{\partial L_{t-1}^k} < 0$) and the lower the level of the spouse's labor supply ($\frac{\partial P_t^k}{\partial L_{t-1}^l} > 0$). The intuition behind $\frac{\partial P_t^k}{\partial L_{t-1}^k} < 0$ is as follows. The addition of work experience from the past leads to a higher wage rate in the present. As the wage rate goes up, the individual's outside option (the lifetime utility in the state of divorce) improves and thereby a higher compatibility value is needed to prevent the marriage from being dissolved. A higher critical compatibility value implies lower likelihood of marital preservation. The sign of $\frac{\partial^2 P_t^k}{\partial L_{t-1}^{k\,2}}$ suggests that the probability of the couple staying together decreases at an increasing rate with respect to one's own labor supply.

A rise in the spouse's number of hours worked has the opposite effect on marriage. As the spouse's wage rate increases with the amount of human capital accumulated, he/she can potentially make a greater contribution to the welfare of the family. As a result, one is less willing to dissolve the marriage and hence $\frac{\partial P_t^k}{\partial L_{t-1}^l} > 0$. The sign of $\frac{\partial^2 P_t^k}{\partial L_{t-1}^{l\,2}}$ implies that the probability of the marriage remaining intact increases at a decreasing rate with respect to the spouse's work hours.

2.5 Effect of Changes in the Probability of Divorce on Labor Supply

Now that half of the causal link between labor supply and marital dissolution has been uncovered, we are ready to address the rest of the mystery: how the expectation of divorce may alter the time-allocation decision. If we maximize equation (2.8), the value function of individual i in the state of marriage in period t, with respect to L^i_t, the person's own labor supply, we get the following:

$$(2.18)\quad \frac{\partial V^i_t}{\partial L^i_t} = \frac{a^i}{L^i_t} - \frac{g^i_t}{g^i_t(H-L^i_t)+g^j_t(H-L^j_t)} + b\{\frac{\partial P}{\partial L^i_t}[V^i_{t+1}(m)-V^i_{t+1}(d)]$$
$$+P(L^i_t)\frac{\partial V^i_{t+1}(m)}{\partial w^i_{t+1}}\frac{\partial w^i_{t+1}}{\partial L^i_t}+[1-P(L^i_t)]\frac{\partial V^i_{t+1}(d)}{\partial w^i_{t+1}}\frac{\partial w^i_{t+1}}{\partial L^i_t}\}$$
$$=0$$

The above first-order condition determines i's optimal choice of labor supply in period t as a function of the belief about j's level of labor supply[16] and the probability of separation in period t+1.

When we differentiate equation (2.8) with respect to w^i_t, person i's wage rate in period t, we obtain the Benveniste-Scheinkman equation in the state of marriage below:

$$(2.19)\quad \frac{\partial V^i_t(m)}{\partial w^i_t} = \frac{a}{w^i_t}$$

Also, if we differentiate the value function of individual i in the state of divorce in period t,[17] we have:

[16] In the equilibrium, i's conjecture about j's level of labor supply corresponds to j's actual level of labor supply.

$$(2.20)\quad \frac{\partial V_t^i(d)}{\partial w_t^i} = \frac{1}{w_t^i}$$

If we move the time period in (2.19) and (2.20) forward by one (from t to t+1) and plug the new equations into (2.18), we get the following Euler equation for individual i:

$$(2.21)\quad \frac{a^i}{L_t^i} - \frac{g_t^i}{g_t^i(H-L_t^i)+g_t^j(H-L_t^j)} + b\{\frac{\P P}{\P L_t^j}[V_{t+1}^i(m)-V_{t+1}^i(d)] + P(L_t^i)\frac{a^i w^{i\,\prime}}{w_{t+1}^i}$$

$$+[1-P(L_t^i)]\frac{w^{i\,\prime}}{w_{t+1}^i}\} = 0$$ [18]

By symmetry, the Euler equation for individual j is:

$$(2.22)\quad \frac{a^j}{L_t^j} - \frac{g_t^j}{g_t^i(H-L_t^i)+g_t^j(H-L_t^j)} + b\{\frac{\P P}{\P L_t^j}[V_{t+1}^j(m)-V_{t+1}^j(d)] + P(L_t^j)\frac{a^j w^{j\,\prime}}{w_{t+1}^j}$$

$$+[1-P(L_t^j)]\frac{a^j w^{j\,\prime}}{w_{t+1}^j}\} = 0$$

In order to find the optimal level of labor supply for each spouse, we need to solve equation (2.21) and (2.22) simultaneously. However, the analytical solution for the

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[17] Once divorced, the person's lifetime maximization problem is:

$$\max_{\{L_t^i,\ldots,L_T^i\}} \sum_{t=t}^{T} [\ln w_t^i + \ln L_t^i + \ln g_t^i + \ln(H-L_t^i)]$$

system of the two Euler equations is rather intractable. Fortunately, the explicit solution for the exact levels of labor supply is not our primary focus. We are more concerned with how the household members may change their time-allocation choices responding to marital instability. In other words, we are interested in the comparative statics $\frac{\partial L_t^i}{\partial E(q_{t+1}^i)}$ and we can find $\frac{\partial L_t^i}{\partial E(q_{t+1}^i)}$ without solving for L_t^i* and L_t^j*.

The expected compatibility value in period t+1, $E(q_{t+1})$, may have an effect on the labor supply functions of both i and j. Hence, we can alternatively express (2.21) and (2.22) as the following:

$$(2.23)\quad \frac{\partial V_t^i(L_t^i(E(q_{t+1})), L_t^j(E(q_{t+1})), E(q_{t+1}))}{\partial L_t^i} = 0$$

$$(2.24)\quad \frac{\partial V_t^j(L_t^i(E(q_{t+1})), L_t^j(E(q_{t+1})), E(q_{t+1}))}{\partial L_t^j} = 0$$

Differentiating (2.23) and (2.24) with respect to $E(q_{t+1})$ yields the system:

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[18] Since $w_{t+1}^i = w_0^i + w^i(L_t^i)$, $\frac{\partial w_{t+1}^i}{\partial L_t^i} = w^{i\prime}$.

$$(2.25)\quad \begin{pmatrix} \dfrac{\partial V_t^i}{\partial L_t^{i\,2}} & \dfrac{\partial V_t^i}{\partial L_t^i \partial L_t^j} \\ \dfrac{\partial V_t^j}{\partial L_t^i \partial L_t^j} & \dfrac{\partial V_t^j}{\partial L_t^{j\,2}} \end{pmatrix} \begin{pmatrix} \dfrac{\partial L_t^i}{\partial E(q_{t+1})} \\ \dfrac{\partial L_t^j}{\partial E(q_{t+1})} \end{pmatrix} = \begin{pmatrix} -\dfrac{\partial^2 V_t^i}{\partial L_t^i \partial E(q_{t+1})} \\ -\dfrac{\partial V_t^j}{\partial L_t^j \partial E(q_{t+1})} \end{pmatrix}$$

Applying Cramer' s rule to (2.25), we have:

$$(2.26)\quad \frac{\P L_t^i}{\P E(q_{t+1})} = \frac{\begin{vmatrix} -\dfrac{\P^2 V_t^i}{\P L_t^i \P E(q_{t+1})} & \dfrac{\P^2 V_t^i}{\P L_t^i \P L_t^j} \\ -\dfrac{\partial^2 V_t^j}{\partial L_t^j \partial E(q_{t+1})} & \dfrac{\P^2 V_t^j}{\P L_t^{j\,2}} \end{vmatrix}}{\begin{vmatrix} \dfrac{\P^2 V_t^i}{\P L_t^{i\,2}} & \dfrac{\P^2 V_t^i}{\P L_t^i \P L_t^j} \\ \dfrac{\P^2 V_t^j}{\P L_t^j \P L_t^i} & \dfrac{\P^2 V_t^j}{\P L_t^{j\,2}} \end{vmatrix}}$$

Examining equation (2.26), it is obvious that we need to find the second derivatives of the value functions to derive the sign of $\dfrac{\partial L_t^i}{\partial E(q_{t+1})}$. We get the following two equations if we differentiate (2.21) with respect to L^i_t and L^j_t, respectively:

$$(2.27)\quad \frac{\P^2 V_t^i}{\P L_t^{i\,2}} = -\frac{a}{L_t^{i\,2}} - \frac{g_t^{i\,2}}{[g_t^i (H - L_t^i) + g_t^j (H - L_t^j)]^2} + b\{\frac{\P^2 P}{\P L_t^{i\,2}}[V_{t+1}^i(m) - V_{t+1}^i(d)]$$

$$+ 2\frac{\P P}{\P L_t^i}(a^i - 1)\frac{w^{i\,'}}{w_{t+1}^i} + [1 - (1 - a^i)P][\frac{w^{i\,''} w_{t+1}^i - (w^{i\,'})^2}{(w_{t+1}^i)^2}]\}$$

$$(2.28)\quad \frac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j} = -\frac{g_t^i\, g_t^j}{[g_t^i(H - L_t^i) + g_t^j(H - L_t^j)]^2} + b\{\frac{\partial P}{\partial L_t^i}\frac{b^i w^{j'}}{w_{t+1}^j} + (a^i - 1)\frac{\partial P}{\partial L_t^j}\frac{w^{i'}}{w_{t+1}^i}\}$$

Similarly, the following two equations are obtained if we differentiate (2.22) with respect to L^j_t and L^i_t, respectively:

$$(2.29)\quad \frac{\partial^2 V_t^j}{\partial L_t^{j\,2}} = -\frac{a^j}{L_t^{j\,2}} - \frac{g_t^{j\,2}}{[g_t^i(H - L_t^i) + g_t^j(H - L_t^j)]^2} + b\{\frac{\partial^2 P}{\partial L_t^{j\,2}}[V_{t+1}^j(m) - V_{t+1}^j(d)]$$

$$+ 2\frac{\partial P}{\partial L_t^j}(a^j - 1)\frac{w^{j'}}{w_{t+1}^j} + [1 - (1 - a^j)P][\frac{w^{j''} w_{t+1}^j - (w^{j'})^2}{(w_{t+1}^j)^2}]\}$$

$$(2.30)\quad \frac{\partial^2 V_t^j}{\partial L_t^j \partial L_t^i} = -\frac{g_t^i\, g_t^j}{[g_t^i(H - L_t^i) + g_t^j(H - L_t^j)]^2} + b\{\frac{\partial P}{\partial L_t^j}\frac{b^j w^{i'}}{w_{t+1}^i} + (a^j - 1)\frac{\partial P}{\partial L_t^i}\frac{w^{j'}}{w_{t+1}^j}\}$$

In Appendix 2.2, we show that under certain conditions, the denominator on the right hand side of equation (2.26) is positive.[19] Assuming these conditions are satisfied, the sign of $\frac{\partial L_t^i}{\partial E(q_{t+1})}$ is determined by the numerator on the right hand side of equation (2.26). The numerator equals to:

$$(2.31)\quad \begin{vmatrix} -\dfrac{\partial^2 V_t^i}{\partial L_t^i \partial E(q_{t+1})} & \dfrac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j} \\ -\dfrac{\partial^2 V_t^j}{\partial L_t^j \partial E(q_{t+1})} & \dfrac{\partial^2 V_t^j}{\partial {L_t^j}^2} \end{vmatrix} = -\frac{\partial P^i}{\partial L_t^i}\frac{\partial^2 V_t^j}{\partial {L_t^j}^2} + \frac{\partial P^j}{\partial L_t^j}\frac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j}$$

From *Result 2.2*, we know that both $\frac{\partial P^i}{\partial L_t^i} < 0$ and $\frac{\partial P^j}{\partial L_t^j} < 0$. Also, closely examining equation (2.26), we deduce that $\frac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j}$ < 0. It follows that the second term in equation (2.31) is positive and the first term is positive if $\frac{\partial^2 V_t^j}{\partial {L_t^j}^2}$ > 0 and negative if $\frac{\partial^2 V_t^j}{\partial {L_t^j}^2}$ < 0. It turns out that the sign of $-\frac{\partial P_t^i}{\partial L_t^i}\frac{\partial^2 V_t^j}{\partial {L_t^j}^2} + \frac{\partial P_t^j}{\partial L_t^j}\frac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j}$ depends on the relative magnitude of $\frac{w^{i\,\prime}}{w_{t+1}^i}$ and $\frac{w^{i\,\prime}}{w_{t+1}^i}$, the rates of return from working (or wage growth) for i and j. There are four scenarios that can potentially arise due the differing magnitudes of $\frac{w^{i\,\prime}}{w_{t+1}^i}$ and $\frac{w^{i\,\prime}}{w_{t+1}^i}$.

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[19] The positive sign of the Hessian matrix $\begin{vmatrix} \dfrac{\partial^2 V_t^i}{\partial {L_t^i}^2} & \dfrac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j} \\ \dfrac{\partial^2 V_t^j}{\partial L_t^j \partial L_t^i} & \dfrac{\partial^2 V_t^j}{\partial {L_t^j}^2} \end{vmatrix}$ is a sufficient condition for local stability of the dynamical system.

Scenario 1: $\frac{w^{i\prime}}{w^{i}_{t+1}}$ is sufficiently small and $\frac{w^{j\prime}}{w^{j}_{t+1}}$ is sufficiently large.

In this case, $\frac{\partial^2 V_t^j}{\partial L_t^{j\,2}} > 0$ and it follows that $\frac{\partial L_t^i}{\partial E(q_{t+1})} > 0$.

Scenario 2: $\frac{w^{i\prime}}{w^{i}_{t+1}}$ is sufficiently large and $\frac{w^{j\prime}}{w^{j}_{t+1}}$ is sufficiently large.

This scenario is ruled out since the sign of the second order condition would be negative and the assumption of stability of the dynamical system would be violated.

Scenario 3: $\frac{w^{i\prime}}{w^{i}_{t+1}}$ is sufficiently small and $\frac{w^{j\prime}}{w^{j}_{t+1}}$ is sufficiently small.

In this case, $\frac{\partial^2 V_t^j}{\partial L_t^{j\,2}} < 0$ and $\left|\frac{\partial^2 V_t^j}{\partial L_t^{j\,2}}\right| > \left|\frac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j}\right|$. Hence, $\frac{\partial L_t^i}{\partial E(q_{t+1})} < 0$.

Scenario 4: $\frac{w^{i\prime}}{w^{i}_{t+1}}$ is sufficiently large and $\frac{w^{j\prime}}{w^{j}_{t+1}}$ is sufficiently small.

In this case, again, $\frac{\partial^2 V_t^j}{\partial L_t^{j\,2}} < 0$ and $\left|\frac{\partial^2 V_t^j}{\partial L_t^{j\,2}}\right| > \left|\frac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j}\right|$. Therefore, $\frac{\partial L_t^i}{\partial E(q_{t+1})} <$ 0.

We can summarize the above four scenarios with *Proposition 2.1.*

Proposition 1**:** Assuming the stability condition of the dynamic system holds, $\frac{\partial L_t^i}{\partial E(q_{t+1})}$ > 0 if $\frac{w^{i\,\prime}}{w_{t+1}^i}$ is sufficiently small and $\frac{w^{j\,\prime}}{w_{t+1}^j}$ is sufficiently large; otherwise, $\frac{\partial L_t^i}{\partial E(q_{t+1})}$ < 0.

Since we are interested in the labor supply of women in this paper, we assign person i to be the wife and person j to be the husband. *Proposition 2.1* states that as compatibility deteriorates (which translates into the increasing probability of divorce), women would decrease labor supply if their wage growth is very low and their husbands' wage growth is very high; otherwise they would increase labor supply. This finding is consistent with the economic theory of divorce offered by Becker, Landes, and Michael (1977). Applying the Coase Theorem, they reason that divorce would take place only if the husband and wife both become better off as a result of marital dissolution. If it is beneficial for one party to stay in the marriage, that person can always compensate for the other's loss as long as the total utilities of the two people when married exceed the total utilities when separated.

Suppose we have a scenario where it is advantageous for the wife to remain married, but not so for the husband. According to Becker, Landes, and Michael, the wife can potentially "bribe" the husband into staying in the marriage given that her gain from marriage is greater than his loss. In the time-allocation framework, the wife can contribute to the welfare of the husband through working in the labor market or at home.

Depending on whether the husband benefits more from the market good or the household good on the margin, the wife may work either more or less in the labor market in order to please the husband. Overall, there are two different effects that may push the labor supply choice of a wife who can extract gains from marriage in opposite directions as the probability of separation rises. The first effect is the well-documented moral hazard problem that leads to an increased number of hours worked for the wife as a self-insurance scheme. The second effect is related to the preservation of marriage that calls for either an increase or a reduction in labor market activities[20]. In Figure 2.2, we depict a possible time line faced by the wife to further illustrate the nature of these two effects.

<u>Figure 2.2:</u> The Time Line Faced by the Wife

0 D_1 D_2 T

The wife's objective is to maximize the present value of her lifetime utility from 0 to T. Suppose that in period D_1-1, the wife anticipates a possible divorce in D_1. We first assume that the husband can gain from reallocation of the wife's time toward the market and that the wife can delay the divorce date to D_2 by concentrating her efforts in the labor market. In this case, both the moral hazard and marital preservation effects would make the woman increase the number of work hours. On the other hand, if the husband prefers to consume more household good, it is then ambiguous whether the

[20] The marital preservation effect does not exist if the woman is better off in the state of divorce.

woman would work more or less in the market since the two effects would diametrically work against each other. If the marital preservation effect outweighs the moral hazard effect, she would lower the level of labor supply and push back the date of divorce to D_2 and thereby receive total utilities of $\sum_{t=D_1}^{D_2} b^t U_t^w(m) + \sum_{t=D_2}^{T} b^t U_t^w(d)$ for the rest of her life. However, if the moral hazard effect dominates, she would raise the level of labor supply and generate utilities of $\sum_{t=D_1}^{T} b^t U_t^w(d)$.

Hence, a wife would respond to marital instability by reducing labor supply only if:

(2.32) $$\sum_{t=D_1}^{D_2} b^t U_t^w(m) + \sum_{t=D_2}^{T} b^t U_t^w(d) > \sum_{t=D_1}^{T} b^t U_t^w(d)$$

The inequality in (2.32) is most likely to be satisfied by the combination of the high female and low male wage growth. The low wage growth for the wife implies that the welfare in the state of divorce will not be greatly enhanced by the increase in labor supply prior to the separation. As a result, $\sum_{t=D_1}^{T} b^t U_t^w(d)$ tends to be small. Also, the high wage growth for the husband signifies large potential benefits from a prolonged marriage from the wife's standpoint and therefore $\sum_{t=D_1}^{D_2} b^t U_t^w(m) + \sum_{t=D_2}^{T} b^t U_t^w(d)$ is expected to be large.

To summarize the economic reasoning behind *Proposition 2.1*: The sum of the moral hazard and marital preservation effects determines how women would respond to instability in marriage. The moral hazard effect is always positive in the sense that it invariably leads to greater levels of labor supply by women as the probabilities of divorce rise. The marital preservation effect, on the other hand, can be either positive or negative since women may either work more or less in the labor market to prolong the marriage. When the wife's wage growth is high and the husband's wage growth is low, both effects would lead the woman to raise labor supply. When both spouses have either high or low wage growth, the marital preservation effect may be negative but is weak and thereby unlikely to outweigh the moral hazard effect. Finally, if the wage growth is low for the wife and high for the husband, the negative marital preservation effect should dominate the positive moral hazard problem and the woman would increase the number of hours worked.

2.6 Implementation of the Theoretical Predictions

In order to verify *Proposition 2.1* empirically, we need to determine what types of wage growth can be considered as "sufficiently high" and "sufficiently low." It seems that any categorization of "sufficiently high" or "sufficiently low" wage growth would be rather arbitrary. Hence, we decide to use the most general classification possible. A person's wage growth is considered to be "sufficiently high" whenever he/she has a higher wage growth than the spouse. By the same token, someone's wage growth is regarded as "sufficiently low" if he/she has a lower wage growth than the spouse.

Proposition 2.2 below states the reformulated hypothesis that we will focus on in the empirical sections.

Proposition 2.2. Responding to the rising likelihood of marital dissolution, women whose wage growth is greater than that of their husbands would increase labor supply and women whose wage growth is less than that of their husbands would decrease labor supply.

Proposition 2.2 establishes testable predictions regarding differences in wage growth, divorce probability, and labor supply. However, before we can proceed with the test, measurement issues associated with wage growth and divorce probability need be addressed.

There does not seem to be a standard method in measuring wage growth in the current literature. Of course we can always rely on the regression approach by estimating the wages of an individual at two different points in time and calculating the percentage change from one period to the next. But, many of the important determinants of one's wage growth, such as ability and work ethics, are unobservable. In light of the difficulties in capturing the idiosyncratic elements of wage growth with the regression approach, we adopt the "brute-force" technique of computing the percentage increase in the actual wages between two consecutive years. There are, however, two complications associated with the brute-force method.

The first complication arises when the respondents in the data set do not work in one or more years. We use a simple example to illustrate the problem of missing data and how we deal with it. Suppose we have data collected on an individual over a span of

3 years. The person's wage growth in any given year t (other than the first year[21]) is calculated as (wage in year t – wage in year t-1) / wage in year t-1. But if the individual chooses not to work, let's say, in year 2, we cannot obtain the wage growth for any of the three years. To get around the problem of incomplete data in this case, we use the wage in year 3 (assuming the number of hours worked in year 3 is positive) as a substitute for the wage in year 2. It follows that the wage growth in year 2 is computed as (wage in year 3 – wage in year 1) / wage in year 1. Once an estimate of the wage growth in year 2 is found, we use it as a proxy for the wage growth in both years 1 and 3.

There is also an inherent selectivity problem in the way we estimate wage growth. Since the percentage of the actual wage gain is used as the proxy for the rate of return from working, we have to exclude couples that have no work history at all from the sample. The selectivity problem, however, is not serious. Only a very small portion of the sample is kept out in this fashion.

Determining the probability of divorce for an individual in a particular year is an even more daunting task than evaluating wage growth. Since the probability of divorce at each point in time is only observable to the couple, the researchers have to infer that probability from the events that take place in the subsequent years. The researchers nonetheless are not able to detect changes in the likelihood of separation over time. Suppose we are attempting to figure out the probability of marital dissolution for a particular couple in 1980. We can come up with a rough measure of the probability depending on whether the couple separated in the few years following 1980. We cannot, however, find how the probability of divorce had changed from 1980 to 1981 from the

[21] We use the wage growth in year 2 as a substitute for the wage growth in year 1.

data. As we will discuss in the later section, this inability to identify changes in the probability of divorce limits the effectiveness of our estimation techniques.

2.7 Data and Preliminary Statistics

To address our question at hand, we need a longitudinal data set that has detailed information on each respondent's marital and work history as well as other demographic characteristics. The National Longitudinal Survey of Youth (NLSY) for years 1979-1993 contains such information. The respondents in the NLSY were all between 14-22 years old in 1979, the year of the initial interview. There were a total of 6,283 female respondents interviewed during the first year of the survey.

We are interested in whether women who have higher wage growth than their husbands respond to the possibility of marital dissolution differently from those who do not. As a first step of the investigation, we examine the changes in labor supply over time for the following four distinct groups: (1) high wage growth women who eventually divorced, (2) low wage growth women who eventually divorced, (3) high wage growth women whose marriages stayed intact in the subsequent years, and (4) low wage growth women whose marriages stayed intact in the subsequent years.[22]

The sample-inclusion criteria in this exercise are as follows. First, the marriage has to last for four years or longer. Second, either the husband or the wife needs to have positive wage growth in at least one year. Third, the wife has to be in her first marriage. Finally, if divorced, the woman must remain unmarried for at least one year. The labor-

[22] The references of "high wage growth" and "low wage growth" are all made relative to the wage growth of husbands.

force participation rates and hours worked for these four groups of women are listed in Tables 2.1-2.4.

Comparing Tables 2.1 and 2.2, we notice that for those women who subsequently divorced and whose wage growth exceeded that of their husbands, the participation rates and the number of hours worked went up by 12% and 289 hours from the fourth year to the year before the separation. In contrast, the participation rates declined by 2% and the number of hours worked rose by 185 for the women who subsequently divorced and who had low wage growth. The double difference in participation rates between the two groups is 14% and the double difference in the number of hours worked is 104. These figures seem to correspond to our theoretical prediction that the moral hazard effect is much more significant for women with higher wage growth than their husband. However, these double differences may be entirely accounted for by the disparity in relative wage growth. That is, we may observe similar differential patterns among the women who did not separate from their husbands. In order to make inferences about the interaction effect between marital instability and relative wage growth, we need to compare the double differences obtained from the divorced women with those of the non-divorced women.

There is one difficulty in making the comparisons between the double differences for the divorced and non-divorced women: we cannot trace the labor supply behavior of the non-divorced women based on the date of dissolution. We resolve this problem by doing the following. First, we calculate the average age for the divorced women at the time of separation and the average divorce age turns out to be around 27. It follows that the average age for each of the four years preceding the split is 23-26. We then find the levels of labor supply for the non-divorced women at the age of 23-26 and come up with the double differences accordingly.

From Tables 2.3 and 2.4, we see that the double differences in the participation rates and the number of hours worked among the non-divorced women are 6.7% and 169 hours. Hence, the differences between the double differences (the triple differences) are 7.3% (t-statistic = 1.35) and -65 hours (t-statistic = -0.58) [23]. The (relative) significance of the triple difference for the participation rates is consistent with the theoretical prediction that wives with different relative wage growth respond differently to marital instability. The sign of the triple difference for the number of hours worked is, however, puzzling.

2.8 Econometric Model

In Tables 2.1 and 2.2, it is shown that in the third and fourth year prior to the split, women with high wage growth actually have lower labor-force participation rates relative to women with low wage growth[24]. The same phenomenon may very well be manifested in the cross-section sample. Hence, in the econometric model, we should focus on the changes in labor supply from one year to the next rather than on the absolute levels of labor supply in a particular year. Ideally, we like to examine how the changes in the probabilities of divorce affect the changes in the participation rates and work hours;

[23] Notice that the sample size is rather small since we only include women whose marriage lasted for at least four year.

[24] Recall that the wage growth in a given year is computed from the actual wages if the person participated in the labor market at the time. However, if the person did not work in the market, the wage growth is estimated from the potential wages. Since the women in our sample are fairly young (average age of 23 in the fourth year prior to the separation), it is possible that many of the

however, as mentioned previously, we can only infer the probabilities of divorce over a period of time from the subsequent marital history, not the true probabilities of disruption at each point in time. As a result of this inability to estimate the changes in the likelihood of separation, we simply use the (constant) probabilities of divorce in the labor supply equations.

Two separate systems of equations are estimated to explain the variations in labor-force participation and work hours across time periods. The first system consists of a multinomial equation indicating how the participation status has changed and a probit equation specifies whether the marriage dissolves subsequently. In the second system, a censored equation for the number of hours worked and a probit equation for marital separation are included.

The first system of equations is specified as below:

$$(2.33) \quad \Pr ob(\Delta L_i = j) = \frac{e^{b_j ' X_i}}{1 + \sum_{k=1}^{4} e^{b_k ' X_i}}$$

where ΔL_i = 1 if individual i doesn't work in both year t-1 and year t
= 2 if individual i doesn't work in year t-1, but works in year t.
= 3 if individual i works in year t-1, but doesn't work in year t.
= 4 if individual i works in both year t-1 and year t.

X_i = a set of demographic and economic variables (includes the variable approximating the probability of divorce).

young women who would have high wage growth were still in the process of completing their education and therefore not working.

(2.34) $\Pr ob(Divorce = 1) = F(ß'Y_i)$

where Divorce = 1 if the individual has a subsequent divorce.
= 0 if the individual does not have a subsequent divorce.

F = the standard normal distribution.

Y_i = another set of demographic and economic variables (includes the variable indicating changes in the participation status).

Since the decision to dissolve marriage and fluctuation in the labor-force participation status may be interdependent, we need to adopt the two-stage estimation to correct for the potential simultaneity problem. In the first stage, reduced-form equations explaining the probability of divorce and the change in the participation status are estimated. In the second stage, the predicted probability calculated from the first stage regression is used to obtain consistent estimates in the structural equations. The religious and marital tenure variables are presumed not to affect labor supply choices, and thereby are excluded from the change in the participation status equation. By excluding the religious and marital tenure variables, we overidentify the change in the labor-force participation equation. The unemployment and health status variables are left out of the divorce equation because they are not believed to affect the probability of marital separation. By excluding the unemployment and health status variables, we also overidentify the divorce equation.

The second system of equations is specified as follows:

(2.35) $\Delta H_i = b'X_i + e_i$

where ΔH_i = the change in the number of hours worked from year t-1 to year t.

X_i = a set of demographic and economic variables (includes the variable approximating the probability of divorce).

e_i = the error term

(2.36) $\Pr ob(Divorce = 1) = F(ß'Z_i)$

where Divorce = 1 if the individual has a subsequent divorce.
= 0 if the individual does not have a subsequent divorce.

F = the standard normal distribution.

Z_i = another set of demographic and economic variables (includes the variable indicating the change in work hours).

Since the "true" changes in the number of hours worked is not observable if the individual chooses not to work in either year t-1 or year t, equation (2.35) is modeled as a Tobit process. Heckman's two-step procedure is applied to preserve linearity of the coefficients. In the first step, an inverse Mills ratio is calculated from a probit equation with the dependent variable in the equation set to 1 if the person works in both year t-1 and year t, and 0 otherwise. In the second step, the inverse Mills ratio is used to estimate the changes in work hours in an ordinary least squares equation.

The endogeneity problem between the probability of divorce and the change in work hours are addressed in the same manner as in the first system of equations.

2.9 Estimation Results

The year 1986 is chosen as the sample year in the cross-section analysis for two reasons. First, all women in the NLSY are at least 21 years of age by 1986, and thus matured enough to make marital decisions. Second, a relatively large sample of subsequently divorced women can be collected. If we have a sample year later than 1986, many of the older women who separated from their husbands early on in the panel would be excluded. The variables used in the regression analysis are briefly described in Table 2.5. Means of these variables for the four distinct groups of women discussed in Section VII are presented in Table 2.6.

A. First System of Equations

We first focus on the results from the system of equations estimating changes in labor-force participation and the probability of divorce. The number of women in each multinomial category is listed in Table 2.7. The reduced-form and structural multinomial logit equations are presented in Table 2.8 and 2.9, respectively. The coefficients for "choice" 1 (not working in both 1985 and 1986) are normalized to zero. The coefficients for the other three "choices" are therefore all relative to the case of non-participation in both years. Our primary concern is with women who were not previously in the labor market, but decided to enter upon learning that their marriages may be in trouble. Hence, we need to focus on the comparison between the choice of not working in both years and the choice of not working in 1985 but working in 1986. The coefficients under the Choice 2 column give us this comparison.[25]

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[25] In the multinomial logit regression, the "marginal effect" of a variable can be found by raising the coefficient to its exponent. The exponentiated value of the coefficient represents the effect of a one-unit change in the variable on the likelihood of the choice relative to the base choice.

From the structural equation that estimates the changes in the participation status (Table 2.9), it seems that age, race, the presence of young children, and fluctuation in unemployment rates have no impact on whether choice 1 or choice 2 is made. The coefficient on education suggests that an additional year of schooling raises the odds of ending up in category 2 (relative to category 1) by 14%. Having bad health that prevents one from working for pay, on the other hand, lowers the likelihood of being in category 2 by 69%. The barely significant coefficient on the husband's income indicates that a $10,000 increase in the husband's income lowers the probability of the wife working in the second year by 17.7%

The three variables that we have particular interests in are wage growth, divorce probability, and the interaction term between wage growth and divorce probability. The insignificance of the coefficient on wage growth tells us that difference in relative wage growth, by itself, is not sufficient to induce women to alter their decisions regarding labor-force participation. As we will see, the effect of the difference in relative wage growth between the wife and husband only becomes significant when it is interacted with the probability of marital dissolution. The extraordinarily large coefficient on the probability of divorce suggests that it is far less likely for someone with lower wage growth than her husband to be in category 2 as the marriage becomes less stable. Combining the coefficients on the probability of divorce and the interaction term suggests that a rise in the likelihood of separation would also reduce the incentive to participate in the labor market for women who have higher wage growth than their husbands. However, the magnitude of the reduction is substantially smaller compared with wives who have relatively lower wage growth.

Contrary to the studies in the past, we discover that an increase in the probability of divorce universally discourages women from participating in the labor market with the negative effect being especially pronounced among those who have lower wage growth than the husbands. Our result weakly supports *Proposition 2* which predicts that women with lower wage growth than their husbands would reduce labor supply as marriage becomes less stable. Although the propensity to increase labor supply by wives who have higher wage growth than their husbands is not confirmed by the data, we do find high wage growth women to be more likely to participate in the labor market relative to those with low wage growth.

The reduced-form and structural probit equations are listed respectively in Tables 2.10 and 2.11. According to Table 2.11, a woman is more likely to have a divorce if she is a Baptist, Catholic, or living in the South. Each year of education also raises the probability of divorce by 1.8%. On the other hand, she is less likely to get separated if she lives in a non-urban area, has a child, or the older she is. Once we hold the age of woman constant, the length of marriage has no effect on the probability of divorce. Probability of choice2 has a significantly positive effect on divorce. This positive coefficient, however, does not necessarily mean that the probability of divorce is raised by participation in the labor market. The probability difference of somebody who chooses not workin g in 1985, but working in 1986, is not relative to not working in both years in the divorce equation. Instead, it is compared to all other possibilities. These possibilities include working in 1985 but not working in 1986, working in both years, as well as not working in both years. The marginal effect of choice 2 indicates that the probability of marital separation would increase by 0.8% as women become 1% more likely to be in category 2.

B. Second System of Equations

We now turn our attention to the system of equations that estimates changes in the number of hours worked and probabilities of divorce. Here, we only focus on the equation explaining changes in work hours. As explained in the previous section, changes in the number of work hours are estimated as a Tobit equation in order to account for the potential variations attributed to women who did not work in both 1985 and 1986. The probit equation that estimates the inverse Mills ratio is presented in Table 2.12. The coefficients for the OLS equation are listed in Table 2.13. Examining Table 2.13, the only variables that are remotely significant are husband's income, the youngest child with age > 3, and divorce probability. On average, a $10,000 increase in the husband's income lowers the wife's annual number of hours worked by 41 hours. If the youngest child in the family is older than 3, the mother works 292 hours more in a year. However, this coefficient is barely significant. A ten-percent increase in the probability of divorce raises the number of hours worked by almost 197 hours. However, the increase in the divorce probability does not have differential effects on women with different relative wage growth.

2.10 Conclusion

An intertemporal Cournot Nash game is constructed to analyze the dynamic interaction between members of the same household with both labor supply and marital dissolution as endogenous choice variables. The theory predicts that given everything else held constant, an increase in the female labor supply makes it more likely for women to dissolve marriage but less so for men. However, as a wife increases her activities in the labor market, she invariably lowers the amount of time used on home production. If her husband benefits more from the household good than from the market good at the margin, the increase in her level of labor supply would diminish the value of marriage to the husband. This insight turns out to be crucial in explaining how women would alter labor market behavior as the likelihood of marital separation goes up.

In hypothesizing the female labor supply response to unstable marriage, we establish an analytical equivalence to the substitution and income effects induced by a change in the price of a commodity. According to the consumer choice theory, the overall consumption of a good may rise or fall as the price changes since the substitution and income effects may conflict with each other. In the case of marital dissolution, women may either increase or decrease the levels of labor supply as the probabilities of divorce escalate depending on the relative magnitudes of the moral hazard and marital preservation effects. When marriage becomes unstable, the moral hazard effect always leads the wife to increase labor supply. However, the marital preservation effect may bring the woman to work more or less in the labor market. It is most likely for the marital preservation effect to be negative and overshadow the moral hazard effect when the wage growth of the wife is small and the wage growth of the husband is large.

Our theory can perhaps help explain the empirical finding (by Johnson and Skinner) that the increases in female labor supply have no effects on the probabilities of marital separation. In the context of the theory, only women with low wage growth would heighten the likelihood of dissolution by working more in the market. However, they would not increase labor supply in the first place due to the fear that they may lose the gains from marriage if devoting less time to the production of the household good.

We also attempt to empirically verify the modified theoretical prediction that women with lower wage growth than their husbands would work less in the market as the probabilities of divorce increase. When we use labor-force participation as an in indicator for labor supply, it is found that as marriage grows unstable, women generally become less likely to join the labor force, and those with low relative wage growth are especially reluctant to do so. However, when the number of work hours is utilized as the proxy for labor supply, we find the probability of divorce raises the number of hours worked. There is, however, no statistically significant difference can be detected between women with different relative wage growth.

Finally, two restrictive assumptions are made in the model. First, no remarriage is allowed; once the decision to separate is consummated, both the man and woman remain single for the rest of their lives. Second, the efficiency parameters in home production are invariant to the amount of time spent on the manufacturing of the household good. If we relax both assumptions, the main results from the model remain valid. As marriages become less stable, women with high wage growth would be even more inclined to increase labor supply since they can improve their positions in the marriage market with higher wage rates. At the same time, women with low wage growth would have more incentives to concentrate on the home production, as the development of domestic skills is their most effective means to attract potential mates in the future.

We do not incorporate the decision to bear children in the model. However, the fertility choice is often determined simultaneously with marital dissolution and labor supply. The natural extension of the theory is to encompass the fertility behavior and analyze how the childbearing decision is carried out in the context of value of marriage.

Table 2.1: Women Who Divorced Subsequently and Whose Wage Growth in the Fourth Year Before the Separation is Greater than that of the Husbands (Group 1).

Year Before Divorce	Age	Participation Rate	Hours Worked	Hours Worked Conditional on Participation
-4	23.5	0.77 (0.42)	1021 (865)	1320
-3	24.5	0.80 (0.40)	1072 (868)	1341
-2	25.5	0.84 (0.37)	1165 (872)	1389
-1	26.5	0.89 (0.31)	1310 (871)	1466
Total change	3	0.12	289	146

Table 2.2: Women Who Divorced Subsequently and Whose Wage Growth in the Fourth Year Before the Separation is Less than that of the Husbands (Group 2).

Year Before Divorce	Age	Participation Rate	Hours Worked	Hours Worked Conditional on Participation
-4	22.9	0.83 (0.37)	1201 (873)	1442
-3	23.9	0.80 (0.40)	1223 (897)	1528
-2	24.9	0.79 (0.41)	1219 (932)	1542
-1	25.9	0.81 (0.39)	1386 (946)	1702
Total Change	3	-0.02	185	260

Table 2.3: Women Who Did Not Divorce Subsequently and Whose Wage Growth at Age 23 is Greater than that of the Husbands (Group 3).

	Age	Participation Rate	Hours Worked	Hours Worked Conditional on Participation
	23	0.76 (0.43)	1008 (856)	1322
	24	0.78 (0.42)	1081 (862)	1388
	25	0.80 (0.40)	1142 (873)	1422
	26	0.80 (0.40)	1165 (942)	1458
Total Change	3	0.04	157	136

Table 2.4: Women Who Did Not Divorce Subsequently and Whose Wage Growth is Less than that of the Husbands (Group 4).

	Age	Participation Rate	Hours Worked	Hours Worked Conditional on Participation
	23	0.77 (0.42)	1120 (873)	1452
	24	0.74 (0.44)	1108 (893)	1488
	25	0.76 (0.43)	1132 (888)	1491
	26	0.74 (0.44)	1108 (889)	1495
Total Change	3	-0.03	-12	43

Table 2.5: Description of Variables

Age	Age of the respondent
Education	Years of Education
Income	Husband's Total Income
Race	Dummy, = 1 if the person is nonwhite
Tenure	Years of marriage
Tenure^2	Tenure Squared
Second	Dummy, = 1 if the person is in second marriage or above
Baptist	Dummy, = 1 if the person is a Baptist
Catholic	Dummy, = 1 if the person is a Catholic
Frequency	Frequency of religious attendance (scaled between 1-6 with 1 = never)
Nonurban	Dummy, = 1 if residence is in an nonurban area
South	Dummy, = 1 if residence is in the South
Kid<3	Dummy, = 1 if the youngest child is less than 3 years old
Kid>3	Dummy, = 1 if the youngest child is more than 3 years old
Participation 85	Dummy, = 1 if working in 1985
Participation 86	Dummy, = 1 if working in 1986
Hours85	Number of hours worked in 1985
Hours86	Number of hours worked in 1986
Bad health	Dummy, = 1 if health would prevent working for pay
Unemployment	Change in Unemployment Rate from 1985 to 1986

Table 2.6: Means of Variables in 1986 by Subsequent Marital Status and Relative Wage Growth

Variable	Group1 (n=115)[a]	Group 2 (n=97)	Group 3 (n=620)	Group 4 (n=766)
Age	24.6	25.27	25.42	25.34
Education	12.14	12.36	12.48	12.28
Income	18,856	17936	19831	18185
Race	0.21	0.24	0.18	0.19
Tenure	5.13	5.27	5.32	5.37
Second	0.06	0.10	0.07	0.05
Baptist	0.31	0.30	0.24	0.26
Catholic	0.33	0.42	0.35	0.36
Frequency	3.08	3.2	3.11	3.19
Nonurban	0.25	0.24	0.2	0.3
South	0.46	0.45	0.39	0.41
Kid<3	0.87	0.80	0.81	0.81
Kid>3	0.11	0.16	0.17	0.17
Participation 85	0.77	0.89	0.79	0.78
Participation 86	0.84	0.88	0.79	0.74
Hours85	1017	1302	1100	1159
Hours86	1080	1331	1106	1139
Bad health	0.026	0.01	0.032	0.028
Unemploy-ment	-0.10%	-0.09%	-0.02%	-0.05%

[a] The sample size is shown is parentheses.

Table 2.7: Number of Women in Each Multinomial Logit Category:

Choice	Frequency	Fraction
1	238	.1489
2	97	.0607
3	121	.0757
4	1142	.7146

where 1 = not working in both 1985 and 1986 (coefficients normalized to zero).
2 = not working in 1985, but working in 1986.
3 = working in 1985, but not working in 1986.
4 = working in both 1985 and 1986.

Table 2.8: First-Stage Multinomial Logit Equation for Changes in Labor-Force Participation[a]

Variable	Choice 2	Choice 3	Choice 4
Constant	-2.4078 (1.17)	-3.7064 (1.91)	-2.218 (1.82)
Age	0.0569 (0.78)	0.1684 (2.58)	0.1155 (2.5)
Education	0.0910 (1.31)	-0.0567 (0.93)	0.2758 (6.31)
Income x 10^4	-0.2033 (1.71)	0.1186 (1.34)	-0.1103 (1.51)
Nonwhite	0.0726 (0.22)	0.0952 (0.31)	-0.1775 (0.88)
Tenure	-0.3625 (1.86)	-0.2395 (1.21)	-0.4587 (3.68)
Tenure^2	0.0186 (1.3)	-0.0014 (0.09)	0.0201 (2.24)
Second	0.1054 (0.21)	-0.7014 (1.44)	-0.5723 (1.70)
Baptist	-0.6507 (1.88)	-0.0754 (0.24)	0.0162 (0.08)
Catholic	-0.4556 (1.53)	-0.2508 (0.9)	0.0487 (0.26)
Frequency	0.0716 (0.97)	-0.042 (0.6)	-0.0104 (0.23)
Nonurban	0.2835 (1)	-0.0002 (0)	-0.1806 (1.01)
South	-0.4298 (1.47)	-0.1 (0.37)	0.1938 (1.11)
Kid<3	0.3865 (0.33)	0.8879 (0.77)	-0.5324 (0.89)
Kid>3	0.4869 (0.41)	0.6317 (0.53)	-0.2013 (0.33)
Bad health	-0.6316 (1.08)	-0.3984 (0.79)	-1.4468 (3.95)
Unemploy-ment	0.4846 (1.3)	0.7998 (2.42)	0.4852 (1.95)
Wage Growth	0.8737 (3.47)	0.0892 (0.39)	0.2162 (1.41)

[a] The t-statistics are shown in parentheses

[b] Wage growth is a dummy variable. Wage growth = 1 if the wife's wage growth is greater than the husband's wage growth, and = 0 otherwise.

Table 2.9: Second-Stage Multinomial Logit Equation for Changes in Labor Participation[a]

Variable	Choice 2	Choice 3	Choice 4
Constant	2.2393 (0.78)	1.3884 (0.53)	-1.3497 (0.83)
Age	-0.0995 (1.34)	-0.0741 (1.11)	-0.02 (0.46)
Education	0.1311 (2.13)	0.025 (0.46)	0.3554 (9.05)
Income x 10^4	-0.1942 (1.67)	0.1189 (1.39)	-0.1043 (1.65)
Nonwhite	0.1251 (0.36)	0.3851 (1.22)	-0.0368 (0.18)
Kid<3	-0.3249 (0.27)	0.2622 (0.22)	-0.653 (1.06)
Kid>3	-0.5683 (0.46)	-0.5106 (0.42)	-0.5812 (0.906)
Bad health	-1.1698 (1.87)	-0.8917 (1.65)	-1.4844 (3.87)
Unemployment	0.4869 (1.32)	0.7536 (2.3)	0.48648 8 (1.98)
Wage growth	-0.4678 (0.57)	0.462 (0.61)	0.3516 (0.71)
Divorce probability	-17.36 (2.71)	-9 (1.82)	-0.888 (0.29)
Growth x probability[b]	13.69 (2.16)	0.1357 (0.03)	-0.5692 (0.17)

[a] The t-statistics are shown in parentheses

[b] Growth x probability is an interaction term between divorce probability and wage growth.

Table 2.10: First-Stage Divorce Probit Equation

Variable	Coefficient (T-Statistic)	Marginal Effect
Constant	0.6337 (1)	0.1325
Age	-0.0747 (3.13)	-0.0156
Education	0.0126 (0.56)	0.0026
Income x10^6	0.1578 (0.05)	0.0333
Nonwhite	0.1259 (1.18)	0.0263
Tenure	-0.04 (0.64)	-0.0084
Tenure^2	0.0057 (1.16)	0.0012
Second	0.3113 (1.81)	0.0651
Baptist	0.1618 (1.49)	0.0338
Catholic	0.2017 (2.02)	0.0422
Frequency	-0.0097 (0.39)	-0.0020
Nonurban	-0.0243 (0.24)	-0.0051
South	0.1017 (1.11)	0.0213
Kid<3	-0.2531 (0.85)	-0.0529
Kid>3	-0.4247 (1.38)	-0.0888
Wage growth	0.2087 (2.58)	0.0437
Bad health	-0.3153 (1.15)	-0.0659
Unemployment	-0.2071 (0.23)	-0.0057

Table 2.11: Second-Stage Divorce Probit Equation

Variable	Coefficient (T-Statistic)	Marginal Effect
Constant	-0.3516 (0.48)	-0.0734
Age	-0.0662 (2.58)	-0.0138
Education	0.0837 (1.78)	0.0175
Income $x10^6$	-0.7223 (0.15)	-0.1507
Nonwhite	0.0375 (0.33)	0.0078
Tenure	-0.0721 (1.04)	-0.0150
$Tenure^2$	0.0075 (1.4)	0.0016
Second	0.1275 (0.69)	0.0266
Baptist	0.3661 (2.88)	0.0764
Catholic	0.3885 (3.23)	0.0810
Frequency	-0.033 (1.23)	-0.0069
Nonurban	-0.179 (1.65)	-0.0373
South	0.2718 (2.49)	0.0567
Kid<3	-0.5425 (1.65)	-0.1132
Kid>3	-0.6195 (1.94)	-0.1292
Probability of Choice 2	4.0197 (3.15)	0.8385
Probability of Choice 3	1.2441 (1.02)	0.2595
Probability of Choice 4	-0.0144 (0.63)	-0.0030

Table 2.12: Probit Equation for Whether A Person Worked in Both 1985 and 1986 [a]

Variable	Coefficient
Constant	-1.6518 (2.14)
Age	0.006 (0.29)
Education	0.1859 (10.38)
Income x 10^4	-5.9920 (2.06)
Nonwhite	-0.0909 (0.94)
Kid<3	-0.3323 (1.12)
Kid>3	-0.1525 (0.49)
Bad health	-0.6336 (3.1)
Wage growth	0.1560 (0.69)
Divorce probability	2.2801 (1.51)
Growth x probability	-1.6047 (1)

Table 2.13: OLS Equation for Changes in Hours Worked

Variable	Coefficient
Constant	-974.34 (0.97)
Age	-6.35 (0.51)
Education	59.77 (1.12)
Income $\times 10^4$	-41.44 (1.73)
Nonwhite	-89.05 (1.48)
Kid<3	-52.98 (0.28)
Kid>3	292.6 (1.66)
Bad health	-419.72 (1.56)
Unemployment	-51.09 (0.91)
Wage growth	147.48 (1.1)
Divorce probability	1967.02 (1.8)
Growth x probability	-1476.26 (1.44)
Inverse Mills Ratio	602.25 (0.96)

[a] The t-statistics are shown in parentheses

Chapter 3: Marital Separation, Childbearing, and Returns from Labor Market

3.1 Introduction

In Chapter 2, we utilized an intertemporal Cournot game to address the issue of labor supply and marital dissolution. In this chapter, we analyze the female fertility response to the increasing probability of divorce by relaxing the assumption of a fixed number of children in the model presented in Chapter 2.

Most of the research that deals with marriage and fertility focuses on how childbearing affects the stability of marriage but ignores the potential influence of marital dissolution on the fertility behavior. There are few studies that address the simultaneous relationship between marital disruption and childbearing. Koo and Janowitz (1983) estimated a simultaneous logit model to examine the probability of divorce and of giving birth in a given period, but no significant causal relationships were discovered in their analysis. Lillard and Waite (1993) estimated a joint hazards model and found that the hazard of disruption has strong negative effects on the hazard of marital childbearing, and this effect appears to be most pronounced among women who have had at least one child.

Lillard and Waite, however, did not include labor supply in their study and thereby overlooked the close ties between marital stability, fertility choice, and labor market activity. By considering the actual or potential labor market opportunities available to each member of the household, we can gain a better understanding about how the decisions regarding childbearing and marital dissolution are formulated.

The rest of this paper is organized as follows. Section 3.2 outlines the model from Chapter 2 and introduces the fertility variable into the framework. Section 3.3 deals with the effect of children on the stability of marriage. Section 3.4 addresses how the possibility of marital separation influences the decision to give birth. Some descriptive statistics are presented in Section 3.5. In Section 3.6, the econometric model used to test the theoretical prediction is explained. In Section 3.7, the restrictions on the sample are stated. The results from estimation are discussed in Section 3.8. Finally, Section 3.8 concludes.

3.2 The Basic Model

In this section, we present the general framework of the theory with an emphasis on the incorporation of the fertility choice. The utility function is assumed to take on the quasi-log-linear form and each individual's static utility maximization problem is as follows:

In the marriage state:

(3.1) max $$U^i(m) = a^i \ln Z^i + b^i \ln Z^j + \ln N + \ln(Q - \bar{Q}) + q$$

subject to
$$Z^i = w^i L^i$$
$$Z^j = w^j L^j$$
$$N = N(n^i, n^j)$$
$$Q = \frac{(g^i R^i + g^j R^j)}{N}$$
$$L^i + R^i = H$$
$$L^j + R^j = H$$

In the divorce state:

(3.2) max $$U^i(d) = \ln Z^i + \ln N + \ln(Q - \bar{Q})$$

subject to

$$Z^i = w^i L^i$$

$$Q = \frac{g^i R^i}{N}$$

$$L^i + R^i = H$$

Where $U^i(m)$ = person i' s utility in the state of marriage

$U^i(d)$ = person i's utility in the state of divorce

Z^k = the market good purchased by person k (where $k \in \{i, j\}$)

N = the number of children

Q = the average quality of children

$\bar{Q}$ = the minimum level of quality per child

L^k = the number of hours worked in the market by person k where

R^k = the time spent on childcare by person k

w^k = the wage rate of person k

g^k = the efficiency parameter in childcare for person k

H = the total amount of time available in each period

a^i = person i's intensity parameter in consuming the market good purchased by i

b^i = person i's intensity parameter in consuming the market good purchased by j

θ = the compatibility parameter between the couple

Each individual derives utilities from children and the market consumption good (Z). Following Becker's approach, children enter the utility function in the forms of both quantity (N) and quality (Q). We assume that the quality of each child in the household is the same and the required minimum level of quality is $\bar{Q}$. Also, the quality of children cannot be "bought" in the market in the sense that it can only be cultivated through parental care. In other words, the child quality is a direct function of the time parents spent with their children. Specifically, the quality of children is equal to the total "effective" time ($\gamma^i R^i + \gamma^j R^j$) the parents devoted to the children divided by the number of children. There is an inherent tradeoff between quantity and quality of children. Parents enjoy greater levels of utility from having more kids, but additional children are likely to result in lower quality per child. Conversely, if the quality of children were highly valued by parents, they would not choose to have a large number of kids.[26]

Children, both in terms of quantity and quality, are pure public goods in the married households. Once divorced, the quantity of children remains a public good as it still appears in the utility function of both man and woman. The quality of children, on the other hand, becomes a private good if separation occurs. In the state of divorce, each parent can only enjoy the quality generated from the contribution made by him or herself. The transformation of the quality of children from being a public good within marriage to a private good outside marriage is meant to capture the loss of economies of scale in childcare caused by divorce.

In the marriage, members of the household essentially formulate their decisions in two stages. In the first stage, they simultaneously make the time-allocation choices between earning wages in the labor market and caring for children at home given the conjecture about what the other person does. Once the amount of time spent on childcare

[26] The way we model the quantity and quality of children preserves the essence of the Becker

is determined, each individual picks the number of children (n^i and n^j) to maximize on the quantity-quality tradeoff in the second stage. The quantity of children that ends up in the household is a joint function of both individuals' choices, i.e., $N = N(n^i, n^j)$ with $N' > 0$. Hence, each individual's static utility maximization problem within marriage can be simply expressed as:

$$(3.3) \qquad \max_{L^i, n^i} U^i(m) = a^i \ln w^i + a^i \ln L^i + b^i \ln w^j + b^i \ln L^j + \ln N(n^i, n^j) + \ln\left(\frac{g^i(H - L^i) + g^j(H - L^j)}{N(n^i, n^j)} - \bar{Q}\right) + q$$

In the model, individuals can only give birth within marriage. After the marriage dissolves, they are not allowed to have additional kids. Each person's static optimization problem in the state of divorce can be written as:

$$(3.4) \qquad \max_{L^j} U^i(d) = \ln w^i + \ln L^i + \ln N + \ln\left(\frac{g^i(H - L^i)}{N} - \bar{Q}\right)$$

To adequately address the effects of marital instability on fertility patterns, we need to develop an intertemporal framework. In the dynamic story, there is a total of T time periods. At the beginning of each period t, q_t, the compatibility between the couple, is revealed. After that, each individual makes the choices in the following order: (1)

framework in that the shadow price of the child quality is proportional to the quantity of children and the shadow price of quantity is positively related to the level of quality.

marital dissolution, (2) labor supply, and (3) fertility.[27] After the initial period 0, the compatibility parameter in each period is $q_t = q_{t-1} + e_t$ where e_t is a random shock to the compatibility and $e_t \sim N(0, \sigma^2)$. The intertemporal maximization problem for each individual within marriage is:

(3.5) max $$E_0\{\sum_{t=1}^{T} b^t U_t^i(L_t^i, n_t^i)\}$$

subject to $$q_t = q_{t-1} + e_t$$

$$N_t = N_{t-1} + N(n_t^i, n_t^j)$$

$$\frac{\partial N}{\partial n_t^i} = \frac{\partial N}{\partial n_t^j}, \quad \frac{\partial N}{\partial n_t^k} > 0, \frac{\partial^2 N}{\partial n_t^{k\,2}} = 0 \quad \forall k \in \{i, j\}$$

$$w_t^i = w_o^i + w(L_{t-1}^i)$$

$$w_t^j = w_o^j + w^j(L_{t-1}^j)$$

$$w_t^{k'} > 0, w_t^{k''} < 0, w_0^k > 0 \ \forall k \in \{i, j\}$$

3.3 Effect of Fertility Choice on the Probability of Divorce

As mentioned in the introduction, there is possibly an interdependent relationship between childbearing and marital dissolution. To fully understand how the incentive to give birth is distorted in a strained marriage, we need to examine the influence of the presence of children on marital stability. The set-up of our framework facilitates the

[27] In the initial period 0, we assume that the couple agrees to enter marriage upon observing the compatibility parameter q_0. We also assume that neither partner has any kids prior to the

analysis of this issue. In the model, the fertility choice is determined at the end of each period and the decision regarding marriage is made at the beginning of each period (right after the observation of the compatibility factor). Hence, we can ascertain the effect of childbearing behavior on marital stability by finding out how the critical compatibility value[28] in period t is altered by an additional child in period t-1.

In order to demonstrate the impact of children on the probability of divorce, we need to do the following: (1) illustrate the concavity of the value function, $V_t^i(q_t)$, (2) characterize $q_t^i *$, the critical compatibility value for person i at period t, (3) determine how $q_t^i *$ changes with n_{t-1}^i and n_{t-1}^j, the "chosen" number of new kids by both spouses in the previous period.

The set of state variables in the dynamic model includes the marital status, the existing number of children, the wage rates, and the compatibility parameter. Each person's value function at the beginning of period t (right after the realization of q_t) is defined as follows:

(3.6) $$V_t^i(m) = \max_{L_t^i} U_t^i(L_t^i) + bE[V_{t+1}^i \mid L_t^i]$$

$$= \max_{L_t^i} a^i \ln w_t^i + a^i \ln L_t^i + b^i \ln w_t^j + b^i \ln L_t^j + \ln N_{t-1}$$

---

marriage. Hence, both spouses devote all of their time to working in the labor market in the initial period.

[28] The critical compatibility value is the minimum level of compatibility that would allow one to stay in the marriage. The critical level of compatibility is individual-specific, not couple-specific.

$$+\ln\left[\frac{g_t^i(H-L_t^i)+g_t^j(H-L_t^j)}{N_{t-1}}-\bar{Q}\right]+q_t$$

$$+b\{P(L_t^i)V_{t+1}^i(m)+[1-P(L_t^i)]V_{t+1}^i(d)\}$$

where b = the discount rate

$E[\cdot]$ = the expectation operator

P = the probability of staying married in period t+1

$V^i_{t+1}(m)$ = the value function at period t+1 if the person is married

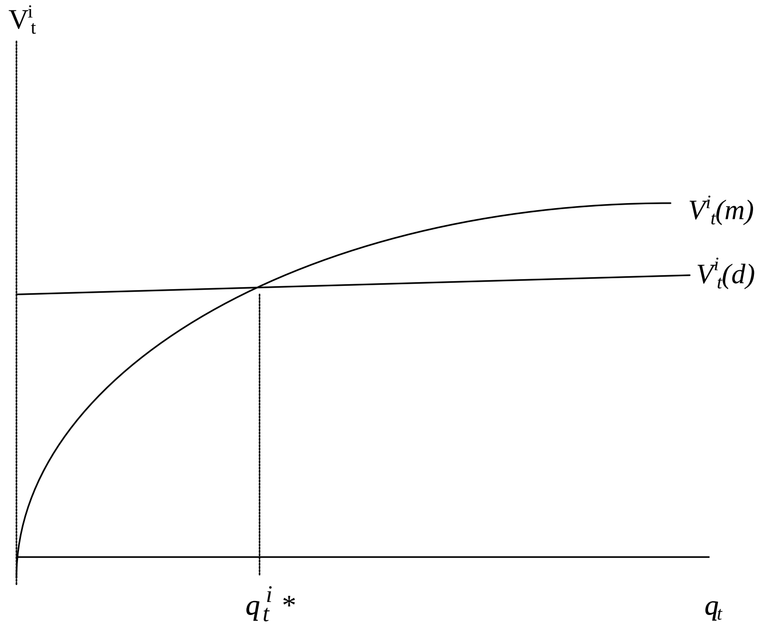
$V^i_{t+1}(d)$ = the value function at period t+1 if the person is divorced

The proof of the concavity of the value function can be found in Chapter 2. Once the concavity of $V^i_t(q_t)$ is established, the optimal policy regarding marital dissolution can be easily shown in Figure 3.1 (on next page).

From Figure 3.1, it is clear that following the optimal dissolution policy, person i would choose to get a divorce in period t if $q_t < q_t^{i*}$, and to stay in the marriage if $q_t \geq q_t^{i*}$. Thus, we can rewrite the value function from equation (3.6) as below:

(3.7) $V^i_t(q_t) = max\{U^i_t(d) + bV^i_{t+1}(d), U^i_t(m) + bV^i_{t+1}(q_{t+1} \mid q_t)\}$

<u>Figure 3.1:</u> Optimal Marital Policy



We now proceed to characterize q^i_t*, the critical compatibility value for person i in period t. From equation (3.7), we know that the if the value of the compatibility parameter equals to q^i_t*, person i would be indifferent between dissolving the marriage or not and the following identity must hold true at the margin:

(3.8) $U^i_t(d) + bV^i_{t+1}(d) = U^i_t(m) + bV^i_{t+1}(q_{t+1} \mid q^i_t*)$

If we plug the expressions from equations (3.3) and (3.4) into equation (3.8), we get the following:

(3.9) $$\ln w_t^i + \ln L_t^i(d) + \ln N_{t-1} + \ln\left(\frac{g_t^i(H - L_t^i(d))}{N_{t-1}} - \bar{Q}\right) + bV_{t+1}^i(d)$$

$$= a^i \ln w_t^i + a^i \ln L_t^i(m) + b^i \ln w_t^j + b^i \ln L_t^j(m) + \ln N_{t-1}$$

$$+ \ln\left[\frac{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))}{N_{t-1}} - \bar{Q}\right] + q_t^i{*} + bV_{t+1}^i(q_{t+1}^i \mid q_t^i{*})$$

$L_t^i(d)$ and $L_t^i(m)$ are the number of hours person i works in the labor market in the state of divorce and the state of marriage respectively. From equation (3.9), $q_t^i{*}$, the critical compatibility level from person i's perspective can be characterized as below:

(3.10) $$q_t^i{*} = \ln w_t^i + \ln L_t^i(d) + \ln N_{t-1} + \ln\left(\frac{g_t^i(H - L_t^i(d))}{N_{t-1}} - \bar{Q}\right) + bV_{t+1}^i(d)$$

$$- a^i \ln w_t^i - a^i \ln L_t^i(m) - b^i \ln w_t^j - b^i \ln L_t^j(m) - \ln N_{t-1}$$

$$+ \ln\left[\frac{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))}{N_{t-1}} - \bar{Q}\right] + q_t^i{*} + bV_{t+1}^i(q_{t+1} \mid q_t^i{*})$$

Differentiating $q_t^i{*}$ with respect to n_{t-1}^i, we get the following:

(3.11) $$\frac{\partial q_t^i{*}}{\partial n_{t-1}^i} = \left[\frac{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))}{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m)) - N_{t-1}\bar{Q}}\right.$$

$$\left. - \frac{g_t^i(H - L_t^i(d))}{g_t^i(H - L_t^i(d)) - N_{t-1}\bar{Q}}\right] \frac{\partial N_{t-1}}{\partial n_{t-1}^i}$$

Unless the amount of time spent on childcare by person i in the state of divorce exceeds that spent by both i and j combined in the state of marriage, i.e., $g_t^i(H - L_t^i(d))$ > $g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^i(m))$, the sign of $\frac{\partial q_t^i *}{\partial n_{t-1}^i}$ is negative. The implication of the negative sign for $\frac{\partial q_t^i *}{\partial n_{t-1}^i}$ is that the probability of staying in the marriage increases with the number of kids, i.e., $\frac{\partial P_t^i}{\partial n_{t-1}^i} > 0$. This result is consistent with numerous empirical studies that found the stabilizing influence of children on marriage. In the context of our model, the rationale is that the loss of economies of scale in childcare caused by marital separation would be greater with more kids. Hence, the incentive to divorce diminishes as the couple has a larger number of children.

If we differentiate $q_t^i *$ with respect to n_{t-1}^j, we find a symmetric result as taking the derivative of $q_t^i *$ with respect to n_{t-1}^i.

(3.12) $$\frac{\partial q_t^i *}{\partial n_{t-1}^j} = [\frac{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))}{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m)) - N_{t-1}\bar{Q}} - \frac{g_t^i(H - L_t^i(d))}{g_t^i(H - L_t^i(d)) - N_{t-1}\bar{Q}}]\frac{\partial N_{t-1}}{\partial n_{t-1}^j}$$

Again, unless $g_t^i(H - L_t^i(d)) > g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^i(m))$, the expression in equation (3.12) should be less than zero. Thus, it is also less likely for the spouse to end the marriage if there are more children in the family. The findings in this section are summarized below by *Result 1* (derivation for the second derivatives of the probability of preserving the marriage is shown in appendix 3.1):

Result 1: $\frac{\partial P_t^k}{\partial n_{t-1}^k} > 0$, $\frac{\partial^2 P_t^k}{\partial n_{t-1}^{k\ 2}} > 0$, $\frac{\partial^2 P_t^k}{\partial n_{t-1}^k \partial n_{t-1}^l} > 0$, $\frac{\partial P_t^k}{\partial n_{t-1}^l} > 0$, $\frac{\partial P_t^{k\,2}}{\partial n_{t-1}^{l\ 2}} > 0$

$\forall k, l \in \{i, j\}$

3.4 Effect of Changes in Probability of Divorce on Fertility Decision

Now we have confirmed that additional children in a family would stabilize the marriage, we can focus on the reverse causal relationship: how does the expectation of marital separation affect the childbearing decision?

In the model, the fertility choices (n_t^i, n_t^j) are made at the end of each period. Hence, the value function of person i with respect to the decision to bear children is:

(3.13) $$V_t^i(m) = \max_{n_t^i} b\{P(n_t^i)V_{t+1}^i(m) + [1 - P(n_t^i)]V_{t+1}^i(d)\}$$

The first-order condition from equation (3.13) is:

(3.14) $$\frac{\partial V_t^i}{\partial n_t^i} = b\{\frac{\partial P}{\partial n_t^i}[V_{t+1}^i(m) - V_{t+1}^i(d)] + P(n_t^i)\frac{\partial V_{t+1}^i(m)}{\partial n_{t+1}^i} + [1 - P(n_t^i)]\frac{\partial V_{t+1}^i(d)}{\partial n_t^i}\}$$

$$= 0$$

Equation (3.14) characterizes $n_t^i *$, the optimal number of children chosen by person i in period t, given the belief about person j's choice and the expected probability of divorce in period t+1.

Differentiating equation (3.6) with respect to n_{t-1}^i, person i's choice with regard to the number of children in period t-1, we have the following Benveniste-Scheinkman equation in the state of marriage:

$$(3.15) \quad \frac{\partial V_t^i(m)}{\partial n_{t-1}^i} = \frac{1}{N_{t-1}} \frac{\partial N_{t-1}}{\partial n_{t-1}^i}[1 - \frac{g_t^i(H - L_t^i) + g_t^j(H - L_t^j)}{g_t^i(H - L_t^i) + g_t^j(H - L_t^j) - N_{t-1}\bar{Q}}]$$

Also, differentiating the value function for individual i in the state of divorce[29], we obtain the following Benveniste- Scheinkman equation:

$$(3.16) \quad \frac{\partial V_t^i(d)}{\partial n_{t-1}^i} = \frac{1}{N_{t-1}} \frac{\partial N_{t-1}}{\partial n_{t-1}^i}[1 - \frac{g_t^i(H - L_t^i)}{g_t^i(H - L_t^i) - N_{t-1}\bar{Q}}]$$

If we push the time period in equation (3.15) and (3.16) forward by one, we have the following:

[29] In the state of divorce, the value function for person i in period t is:

$$V_t^i(d) = \max_{L_t^i} \ln w_t^i + \ln L_t^i + \ln N_{t-1} + \ln[\frac{g_t^i(H - L_t^i)}{N_{t-1}} - \bar{Q}] + bV_{t+1}^i(d|\ L_t^i)$$

$$\text{(3.17)} \quad \frac{\partial V_{t+1}^{i}(m)}{\partial n_t^i} = \frac{1}{N_t}\frac{\partial N_t}{\partial n_t^i}[1-\frac{g_{\ t+1}^{i}(H-L_{t+1}^{i})+g_{\ t+1}^{j}(H-L_{t+1}^{j})}{g_{\ t+1}^{i}(H-L_{t+1}^{i})+g_{\ t+1}^{j}(H-L_{t+1}^{j})-N_t\bar{Q}}]$$

$$\text{(3.18)} \quad \frac{\partial V_{t+1}^{i}(d)}{\partial n_t^i} = \frac{1}{N_t}\frac{\partial N_t}{\partial n_t^i}[1-\frac{g_{\ t+1}^{i}(H-L_{t+1}^{i})}{g_{\ t+1}^{i}(H-L_{t+1}^{i})-N_t\bar{Q}}]$$

We plug equations (3.17) and (3.18) into equation (3.14), we get the following Euler equation for person i:

$$\text{(3.19)} \quad b\{\frac{\partial P}{\partial n_t^i}[V_{t+1}^{i}(m)-V_{t+1}^{i}(d)]$$

$$+P(n_t^i)\frac{1}{N_t}\frac{\partial N_t}{\partial n_t^i}[1-\frac{g_{\ t+1}^{i}(H-L_{t+1}^{i})+g_{\ t+1}^{j}(H-L_{t+1}^{j})}{g_{\ t+1}^{i}(H-L_{t+1}^{i})+g_{\ t+1}^{j}(H-L_{t+1}^{j})-N_t\bar{Q}}]$$

$$+[1-P(n_t^i)]\frac{1}{N_t}\frac{\partial N_t}{\partial n_t^i}[1-\frac{g_{\ t+1}^{i}(H-L_{t+1}^{i})}{g_{\ t+1}^{i}(H-L_{t+1}^{i})-N_t\bar{Q}}]\}$$

$$= 0$$

By symmetry, the Euler equation for person j is:

$$\text{(3.20)} \quad b\{\frac{\partial P}{\partial n_t^j}[V_{t+1}^{j}(m)-V_{t+1}^{j}(d)]$$

$$+P(n_t^j)\frac{1}{N_t}\frac{\partial N_t}{\partial n_t^j}[1-\frac{g_{\ t+1}^{i}(H-L_{t+1}^{i})+g_{\ t+1}^{j}(H-L_{t+1}^{j})}{g_{\ t+1}^{i}(H-L_{t+1}^{i})+g_{\ t+1}^{j}(H-L_{t+1}^{j})-N_t\bar{Q}}]$$

$$+[1-P(n_t^j)]\frac{1}{N_t}\frac{\partial N_t}{\partial n_t^j}[1-\frac{g_{t+1}^j(H-L_{t+1}^j)}{g_{t+1}^j(H-L_{t+1}^j)-N_t\bar{Q}}]\}$$

$$=0$$

The system of Euler equations above (3.19 and 3.20) characterizes the Cournot solution for the optimal number of new children in period t. From equation (3.19) and (3.20), we can proceed to address how changes in the expected compatibility may sway the current fertility choice, i.e., what is the sign of $\frac{\partial n_t^i}{\partial E(q_{t+1})}$.

The expected compatibility parameter in period t+1, $E(q_{t+1})$, has an effect on the optimal choice of new kids in period t for both person i and person j. We can alternatively express the conditions for the Cournot-Nash equilibrium as:

$$\frac{\partial V_t^i(L_t^i(E(q_{t+1})),L_t^j(E(q_{t+1})),E(q_{t+1}))}{\partial n_t^i}=0 \tag{3.21}$$

$$\frac{\partial V_t^j(L_t^i(E(q_{t+1})),L_t^j(E(q_{t+1})),E(q_{t+1}))}{\partial n_t^j}=0 \tag{3.22}$$

Differentiating equation (3.21) and (3.22) with respect to $E(q_{t+1})$ gives us the system:

(3.23)
$$\begin{pmatrix} \dfrac{\partial V_t^i}{\partial n_t^{i\,2}} & \dfrac{\partial V_t^i}{\partial n_t^i \partial n_t^j} \\ \dfrac{\partial V_t^j}{\partial n_t^i \partial n_t^j} & \dfrac{\partial V_t^j}{\partial n_t^{j\,2}} \end{pmatrix} \begin{pmatrix} \dfrac{\partial n_t^i}{\partial E(q_{t+1})} \\ \dfrac{\partial n_t^j}{\partial E(q_{t+1})} \end{pmatrix} = \begin{pmatrix} -\dfrac{\partial^2 V_t^i}{\partial n_t^i \partial E(q_{t+1})} \\ -\dfrac{\partial V_t^j}{\partial n_t^j \partial E(q_{t+1})} \end{pmatrix}$$

Applying Cramer's rule to equation (3.23), we have the following:

(3.24)
$$\frac{\partial n_t^i}{\partial E(q_{t+1})} = \frac{\begin{vmatrix} -\dfrac{\partial^2 V_t^i}{\partial n_t^i \partial E(q_{t+1})} & \dfrac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j} \\ -\dfrac{\partial^2 V_t^j}{\partial n_t^j \partial E(q_{t+1})} & \dfrac{\partial^2 V_t^j}{\partial n_t^{j\,2}} \end{vmatrix}}{\begin{vmatrix} \dfrac{\partial^2 V_t^i}{\partial n_t^{i\,2}} & \dfrac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j} \\ \dfrac{\partial^2 V_t^j}{\partial n_t^j \partial n_t^i} & \dfrac{\partial^2 V_t^j}{\partial n_t^{j\,2}} \end{vmatrix}}$$

If we assume that the system (equations 3.21 & 3.22) is stable, then the value of the denominator in equation (3.24) is greater than 0. It follows that the sign of $\dfrac{\partial n_t^i}{\partial E(q_{t+1})}$ depends on the sign of the numerator in equation (3.24). The numerator is equal to:

(3.25)
$$\begin{vmatrix} -\dfrac{\partial^2 V_t^i}{\partial n_t^i \partial E(q_{t+1})} & \dfrac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j} \\ -\dfrac{\partial^2 V_t^j}{\partial n_t^j \partial E(q_{t+1})} & \dfrac{\partial^2 V_t^j}{\partial n_t^{j\,2}} \end{vmatrix} = -\frac{\partial P_t^i}{\partial n_t^i}\frac{\partial^2 V_t^j}{\partial n_t^{j\,2}} + \frac{\partial P_t^j}{\partial n_t^j}\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$$

From equation (3.25), we see that the sign of $\frac{\partial n_t^i}{\partial E(q_{t+1})}$ depends on the relative magnitude of $\frac{\partial P_t^i}{\partial n_t^i}\frac{\partial^2 V_t^j}{\partial {n_t^j}^2}$ and $\frac{\partial P_t^j}{\partial n_t^j}\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$. The sign of $\frac{\partial n_t^i}{\partial E(q_{t+1})}$ is negative if $\frac{\partial P_t^i}{\partial n_t^i}\frac{\partial^2 V_t^j}{\partial {n_t^j}^2} > \frac{\partial P_t^j}{\partial n_t^j}\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$ and positive if $\frac{\partial P_t^i}{\partial n_t^i}\frac{\partial^2 V_t^j}{\partial {n_t^j}^2} < \frac{\partial P_t^j}{\partial n_t^j}\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$. The values for $\frac{\partial P_t^i}{\partial n_t^i}$ and $\frac{\partial P_t^j}{\partial n_t^j}$ can be found in Appendix 3.1. The values for $\frac{\partial^2 V_t^j}{\partial {n_t^j}^2}$ and $\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$ are listed below:

(3.26)
$$\frac{\partial^2 V_t^j}{\partial {n_t^j}^2} = b\{\frac{\partial^2 P_t^j}{\partial {n_t^j}^2}[V_{t+1}^j(m) - V_{t+1}^j(d)] + \frac{\partial P_t^j}{\partial n_t^j}[\frac{\partial V_{t+1}^j(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^j(d)}{\partial n_t^j}]$$
$$+ \frac{\partial P_t^j}{\partial n_t^j}[\frac{\partial V_{t+1}^j(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^j(d)}{\partial n_t^j}] + P_t^j \frac{\partial^2 V_{t+1}^j(m)}{\partial {n_t^j}^2} + (1 - P_t^j)\frac{\partial^2 V_{t+1}^j(d)}{\partial {n_t^j}^2}\}$$

(3.27)
$$\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j} = b\{\frac{\partial^2 P_t^i}{\partial n_t^i \partial n_t^j}[V_{t+1}^i(m) - V_{t+1}^i(d)] + \frac{\partial P_t^i}{\partial n_t^i}[\frac{\partial V_{t+1}^i(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^i(d)}{\partial n_t^j}]$$
$$+ \frac{\partial P_t^i}{\partial n_t^j}[\frac{\partial V_{t+1}^i(m)}{\partial n_t^i} - \frac{\partial V_{t+1}^i(d)}{\partial n_t^i}] + P_t^i \frac{\partial^2 V_{t+1}^i(m)}{\partial n_t^i \partial n_t^j} + (1 - P_t^i)\frac{\partial^2 V_{t+1}^i(d)}{\partial n_t^i \partial n_t^j}\}$$

Due to the complexity of the two expressions above, it is difficult to directly determine the relative magnitudes of $\frac{\partial P_t^i}{\partial n_t^i}\frac{\partial^2 V_t^j}{\partial {n_t^j}^2}$ and $\frac{\partial P_t^j}{\partial n_t^j}\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$. However, we can solve this problem by breaking down the equations and comparing them by parts. We first compare the relative magnitude of $\frac{\partial^2 V_t^j}{\partial n_t^{j\,2}}$ and $\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$. We denote the difference between the ith term on the right-hand side of (3.26) and the ith term on the right-hand side of (3.27) as Di. That is, D1 is the difference between the first term in (3.26) and the first term in (3.27) and we express D1 in the following equation:

(3.28) $$\text{D1} = \frac{\partial^2 P_t^j}{\partial n_t^{j\,2}}[V_{t+1}^j(m) - V_{t+1}^j(d)] - \frac{\partial^2 P_t^i}{\partial n_t^i \partial n_t^j}[V_{t+1}^i(m) - V_{t+1}^i(d)].$$

It follows that:

(3.29) $$\text{D2} = \frac{\partial P_t^j}{\partial n_t^j}[\frac{\partial V_{t+1}^j(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^j(d)}{\partial n_t^j}] - \frac{\partial P_t^i}{\partial n_t^i}[\frac{\partial V_{t+1}^i(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^i(d)}{\partial n_t^j}]$$

(3.30) $$\text{D3} = \frac{\partial P_t^j}{\partial n_t^j}[\frac{\partial V_{t+1}^j(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^j(d)}{\partial n_t^j}] - \frac{\partial P_t^i}{\partial n_t^j}[\frac{\partial V_{t+1}^i(m)}{\partial n_t^i} - \frac{\partial V_{t+1}^i(d)}{\partial n_t^i}]$$

(3.31) $$\text{D4} = P_t^j \frac{\partial^2 V_{t+1}^j(m)}{\partial n_t^{j\,2}} - P_t^i \frac{\partial^2 V_{t+1}^i(m)}{\partial n_t^i \partial n_t^j}$$

(3.32) $$\text{D5} = (1 - P_t^j)\frac{\partial^2 V_{t+1}^j(d)}{\partial n_t^{j\,2}} - (1 - P_t^i)\frac{\partial^2 V_{t+1}^i(d)}{\partial n_t^i \partial n_t^j}$$

The details of each comparison are shown in Appendix 3.2. It turns out that D1 - D5 all are greater than zero if person i has a higher wage rate than person j. Hence, $\frac{\partial^2 V_t^j}{\partial n_t^{j\,2}} > \frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$ as long as person i' has greater earning potential than person j. However, we are still unsure about the relative sizes of $\frac{\partial P_t^i}{\partial n_t^i}$ and $\frac{\partial P_t^j}{\partial n_t^j}$. From Appendix 3.1, we can infer that $\frac{\partial P_t^i}{\partial n_t^i} < \frac{\partial P_t^j}{\partial n_t^j}$ when there is a wage disparity in favor of person i, unless $N_{t-1} = 0$. If N_{t-1} is not equal to zero, it is ambiguous whether $\frac{\partial n_t^i}{\partial E(q_{t+1})} = -\frac{\partial P_t^i}{\partial n_t^i}\frac{\partial^2 V_t^j}{\partial n_t^{j\,2}} + \frac{\partial P_t^j}{\partial n_t^j}\frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$ is positive or negative since we have $\frac{\partial P_t^i}{\partial n_t^i} < \frac{\partial P_t^j}{\partial n_t^j}$ and $\frac{\partial^2 V_t^j}{\partial n_t^{j\,2}} > \frac{\partial^2 V_t^i}{\partial n_t^i \partial n_t^j}$ (with the assumption of a higher wage rate for person i). On the other hand, if N_{t-1} is equal to zero then $\frac{\partial P_t^i}{\partial n_t^i} = \frac{\partial P_t^j}{\partial n_t^j}$ and consequently $\frac{\partial n_t^i}{\partial E(q_{t+1})} < 0$ (again, with the assumption that person i has a higher wage rate). Denoting person i as the wife and person j as the husband, we summarize the implications derived in this section by *Result 2.*

Result 2: Responding to marital instability, childless wives would be more likely to give birth if they have higher wage rates than their husbands.

The intuition behind *Result 2* is as follows. In the model, each person can marry only once and is not allowed to bear children outside marriage. In addition, individuals derive utilities from children as well as from the market consumption good. As the likelihood of separation becomes greater, a woman may want to have (more) kids as she gains satisfaction from them. If the marginal utility obtained from having children is diminishing, the desire to bear children should be especially strong for women who do not have any kids. [30] Raising children, however, may take a great deal of time away from her labor market activities which in turn leads to a lower wage and less consumption of the market good in the possible state of divorce if her wage rate is lower than that of the husband. If the wife has a higher wage rate than the husband, on the other hand, she would not have to devote as much time to childcare since her comparative advantage lies within the labor market. Hence, childless women with higher wage rates than their husbands are most likely to bear children as marriage becomes less stable.

3.5 Descriptive Statistics

In the rest of this paper, we attempt to verify the theoretical prediction that women without prior children are more likely to give birth as a response to marital instability if they have higher wage rates than their husbands.

The data set used to test the theory is the National Longitudinal Survey of Youth 1979 (NLSY79) that followed approximately 12,000 young men and women (who were

[30] On the other hand, there should be a lack of desire to bear children as the likelihood of dissolution rises for women who already have children. Lillard and Waite showed that is indeed the case in their paper.

14-22 years old when they were first surveyed in 1979) from 1979 to 1998. From the panel, we find that women on average divorced at the age of 28. We compare the fertility behavior of divorced women during the last two years of marriage with that of non-divorced women between age 26-28 based on the relative wage rates to the husbands and whether or not they have prior children. Table 3.1 and 3.2 are based on whether women have higher wage rates than their husbands three years prior to the separation (for the divorcees) or at the age of 25 (for the non-divorcees). Table 3.3 and 3.4 are based on if wives have wage rates at least 1.5 times as high as husbands' wage rates. Finally, Table 3.5 and 3.6 are based on whether wives have wages at least twice as high as husbands' wages.

Examining Tables 3.1 and 3.2, the possibility of separation has roughly the same effect on childbearing between the high-wage and low-wage (relative to the husbands) women. However, as a response to divorce, women with prior kids are less likely to give birth while women without prior kids are more likely to give birth. This observation seems to confirm our intuition that women who do not have prior kids may want to bear children as the probability of marital dissolution increases.

In Tables 3.3 and 3.4, the ratio of wage differential between wife and husband is raised to 1.5. From Table 3.3, we see that among the women who have prior children, those whose wages are not 1.5 times greater than those of their husbands are much less likely to give birth compared with those whose wages are at least 1.5 times higher. For the low-wage women, the proportion that gave birth declines from 46.3% to 32.6%. For the high-wage women, the fraction is lowered from 52.3% to 46.8%. The double difference is 8.2% with a t-statistic of approximately 1. This preliminary finding suggests that marital separation may have differential effects on the fertility behavior depending on the relative wage rates. Turning attention to Table 3.4, among those without prior children, there is not a striking difference between the high-wage and low-wage women in terms of childbearing behavior. The double difference is only 2.2%.

The ratio of wage differential is further increased to 2 in Tables 3.5 and 3.6. Table 3.5 tells us a similar story as Table 3.3 in that for women who have prior kids, those without significantly higher wages than the husbands are less likely to bear children as the likelihood of divorce rises. Table 3.6 kind of confirms the theoretical prediction that the likelihood to bear children for childless women on the brink of marital dissolution is greater if they have higher wage rates than their husbands. Responding to divorce, the birth rate for women who have wages at least twice as high as those of their spouses rises from 9.3% to 17.35% while the birth rate for women whose wages are not twice as high only increases from 10.8% to 13.6%. This accounts for a double difference of 5.2% with a t-statistic of 1.16.

The general patterns from Tables 3.1-3.6 (especially Table 3.6) seem to support our hypothesis. However, we still need to test the theory in a more rigorous fashion. The formal econometric model used for testing is introduced in the next section.

3.6 Econometric Model

As shown in the theoretical model, the decisions to bear children and to dissolve marriage are interdependent. When we estimate how the fertility patterns may be altered by the prospect of a divorce, we also need to account for the potential effect of additional children on marital stability. Hence, we need to estimate a system of two equations with the divorce and fertility variables being endogenous. This system of equations is specified as below:

(3.33) $\text{Prob}(\text{Divorce} = 1) = F(\text{ß}_1{}'X_i)$

Where Divorce = 1 if the respondent divorced between 1993-1998.

= 0 if the respondent didn't divorce between 1993-1998

F = the standard normal distribution.

X_i = a set of demographic and economic variables (including the fertility variable indicating whether the respondent gave birth between 1990-1992).

(3.34) $\text{Prob}(\text{Birth} = 1) = F(\text{ß}_2{}'Y_i)$

Where Birth = 1 if the respondent gave birth between 1990-1992.

= 0 if the respondent didn't give birth between 1990-1992.

Y_i = another set of demographic and economic variables (including the variable specifying if the respondent has a subsequent divorce from 1993-1998).[31]

Consistent estimation of the above system of equations would simultaneously reveal whether the birth of a child between 1990 and 1992 affects the (observed) subsequent marital stability between 1993 and 1998 and whether the expectation of marital dissolution influences the decision to give birth between 1990 and 1992.

[31] The descriptions of all the variables in the regressions are listed in Table 1.

A two-stage method is used to estimate the system of probit equations. In the first stage, both the divorce and birth variables are regressed against all the exogenous variables in the system, $\{X \cap Y\}$. In the second stage, the predicted probabilities of childbirth and divorce, computed from the first-stage estimates, are then used as regressors in equation (3.33) and (3.34) respectively. The divorce equation is overidentified by the exclusion of the variables regarding wages, the number of siblings, and whether there are kids in the household before 1990. The childbirth equation is identified by the variable indicating whether the couple lived together prior to the marriage.[32]

3.7 Description of the Sample

The year 1992 is selected as the sample year for the cross-section regression analysis for two reasons. First, almost all the respondents in the sample had reached the marriage age by 1992. Second, there are still enough years left in the panel to allow the observation of the subsequent marital history, i.e., whether the respondent had a divorce.

Two additional restrictions are placed on the cross-section sample. First, either the husband or the wife must have a valid wage rate (a wage greater than $0 and less than $200) in at least one year from 1989 to 1998. The measure of wage disparities between husband and wife in the regressions is based on the wages in 1989[33], the year of conception for a child born in 1990. The wage rate for a given year is calculated as the

[32] A survey conducted by the Centers for Disease Control and Prevention found that couples who had lived together before marriage are more likely to get divorced than those who hadn't live together at first.

total earned income divided by the number of hours worked in that year. If the wage data is invalid for a particular woman in 1989, we substitute it with her/his wage in 1990. If the wage data for the year 1990 is also invalid, we use the wage in 1991 as a proxy for the wage in 1989, so on and so forth. In the case when the woman does not have a valid wage from 1989 to 1998, we denote her wage in 1989 as zero. However, if her spouse also lacks a valid wage during that span, she is excluded from the sample.

Second, only women whose marriages last for at least four years (before 1992) are included in the sample. This condition is in place to ensure that all couples had been married for at least one year prior to the conception of a child who was born between 1990 and 1992.

Overall, there are 1,869 couples who had valid wage rates and had been married for a minimum of four years.

3.8 Estimation Results

Before proceeding to discuss the results of estimation, we need to address how to explicitly test the hypothesis in the two-stage framework.

The theory predicts that childless women who have higher wages than their husbands are likely to give birth as their marriages destabilize. We detect preliminary evidence of that from Table 3.6 when we increase the ratio of wage differential between wife and husband to 2. From the cross-section sample shown in Table 3.8, we see that in 1989, 31.6% of the married women have higher wage rates than their husbands (dwage189), 17.4% have wages at least 1.5 times higher (dwage289), and 13.2% have

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[33] Suppose a child was born in 1990, the decision to become pregnant was most likely made in

wages at least twice as high (dwage389). Initially, we create dummy variables indicating whether the woman has a wage rate that is higher, 1.5 times higher, or 2 times higher than the husband's wage rate to represent wage disparities in the households. These dummies are interacted with the predicted probability of divorce estimated from the first stage and the dummy variable indicating whether there are no prior kids in the family (i.e., dwage189 $\times$ predicted probability of divorce $\times$ noprekid, etc) to test the validity of the theory in the second-stage childbirth equation. However, none of these wage disparity variables and interaction terms turned out to be significant. The regression results with those wage-disparity dummies are not included in this paper.

After failing to obtain significant results with the wage-differential dummies, we simply express the wage disparity within the household as a continuous variable. In the unconstrained version of the estimation, the wife's wage and the husband's wage are regressed as two separate variables in the equation. In the constrained version, the difference between the wife's wage and the husband's wage is regressed. The results from the unconstrained model are presented in Tables 3.9-3.12 and the results from the constrained model are shown in Tables 3.13-3.16.

A. The Unconstrained Model

In the unconstrained model, the respondent's wage (Owage89) and the spouse's wage (Swage89) are regressed separately. The reduced-form and structural probit equations for marital dissolution are presented in Tables 3.9 and 3.10 respectively. Examining the structural equation for divorce, we see that as a woman becomes one year

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1989. Hence, the wage rate in 1989 instead of 1990 is used as a determinant of childbirth.

older (age92), she is1.9 percent less likely to get divorced.[34] The age of the husband, on the other hand, has no significant effect on the probability of divorce. At the same time, each additional year of marital tenure (tenure) lowers the likelihood of the split at a constant rate (the square term of the tenure variable is insignificant) of 3%. Together, this means that as each calendar year passes, the couple becomes almost 5% less likely to separate. The probability of dissolution is also lowered if someone is black (3.7%) and lives in a non-urban area (3.5%), however these effects are not highly significant. Interestingly, the coefficient for the before variable seems to confirm the study done by the Center for Diseases Control and Prevention that couples who lived together before marriage are (4.6%) more likely to end up in divorce. Finally, a 1% increase in the likelihood of childbirth between 1990 and 1992 lowers the probability of divorce from 1993 to 1998 by 0.6%. This finding is consistent with the prediction of *Result 1*.

We now shift the focus to the regression for childbirth. The reduced-form and structural probit equations for childbirth are presented in Table 3.11 and 3.12 respectively. In the second-stage childbirth regression, we include several interaction terms to directly test *Result 2*. Inter1 is the interaction between the wife's wage and the predicted probability of divorce. Inter2 is the interaction between the husband's wage and the predicted probability of divorce. Inter3 is the interaction term between the wife's wage and a dummy variable indicating whether there are no children in the household before 1990 (noprekid). Inter4 is the interaction term between the husband's wage and the dummy specifying the absence of prior kids. Inter 5 is the interaction term between

[34] It is possible that the older women in the sample are more old-fashioned and “value” marriage more than the younger cohorts do. In that case, the coefficient on the age variable may reflect the cohort effect.

the absence of prior kids and the predicted probability of divorce. Inter6 is the interaction term between the wife's wage, the predicted probability of divorce, and the absence of prior kids. Finally, Inter7 is the interaction term between the husband's wage, the predicted probability of divorce, and the absence of prior kids.

Examining the second-stage equation for childbirth has uncovered a few surprising results. First, black women and women who have residence in the south are 9.8% and 4.5% less likely to give birth, respectively. Second, marital tenure seems to have a nonlinear negative effect on childbearing. Third, the odds of giving birth are reduced by a whopping 39.29% for women who do not already have kids by 1990. At last, we turn our attention to testing the theoretical prediction that the childless women who have higher wage rates than their spouses are likely to give birth as a response to the rising probability of divorce. We can verify the hypothesis by checking the statistical significance of $ß_{Inter6} - ß_{Inter7}$. However, the t-statistic for $ß_{Inter6} - ß_{Inter7}$ turns out to be insignificant with an absolute value of 0.34.

B. The Constrained Model

In the constrained model, instead of regressing the wife's wage and the husband's wage separately, we use the difference (that is, difference = wife's wage - husband's wage) in the equation. The reduced-form and structural probit equations for marital dissolution are presented in Tables 3.13 and 3.14 respectively. In the constrained model, black women and women who live in the non-urban areas are no longer less likely to dissolve marriage (the coefficients have become less significant). The probability of divorce declines by 1.9% and 2.8% respectively as the wife's age and marital tenure increase by one year. Hence, each additional calendar year lowers the likelihood of the split by 4.7%. Living together before marriage raises the chance of divorce by 5%. As the probability of giving birth between 1990 and 1992 goes up by 1%, the probability of subsequent dissolution decreases by 0.5% and thereby *Result 1* is confirmed once again.

The reduced-form and structural probit equations for childbirth are presented in Tables 3.15 and 3.16 respectively. In the second-stage childbirth equation, there are four interaction terms included. Here, Inter1 is the interaction term between the difference in wages and the predicted probability of divorce. Inter2 is the interaction between the difference in wages and the dummy variable indicating the absence of previous kids. Inter3 is the interaction between the absence of prior kid dummy and the predicted probability of divorce. Finally, Inter4 is the interaction between the wage difference, the absence of prior kid dummy, and the predicted probability of divorce.

Examining the structural childbirth equation in Table 3.16, we see that black women, women who live in rural areas, and women who reside in the south are respectively 10.3%, 4.9%, and 4.9% less likely to give birth between 1990 and 1992. Also, a 1% increase in the predicted probability of divorce lowers the likelihood of childbirth by 0.6%. In order to test *Result 2* in the constrained model, we need to examine the significance of the coefficient on Inter4, the interaction term between the difference in wages, the childless dummy, and the probability of divorce. As we can see, Inter4 is close to being significant at the 5% level with a t-statistic of 1.85. The marginal effect of Inter4 on the probability of childbirth is about 0.06. This signifies that a one-percent increase in the probability of divorce, coupled with a $1 increase in the wage difference would raise the probability of bearing children by 0.06% for childless women. This finding is consistent with our theoretical prediction.

3.9 Conclusion

Once a couple decides to form a family, they invariably make investments in the marriage. Perhaps the most important and everlasting form of investment one can make within marriage is in the children. The fertility choice, like many other decisions in marriage, is made jointly by husband and wife. Economists generally assume that the process through which the fertility and other choices are determined within a family is cooperative in nature when the marriage is harmonious. In good times, decisions are formulated to optimize the welfare of the family as a whole. However, when the marriage is in trouble, cooperative bargaining may break down and the resulting equilibrium may become one of non-cooperation.

In this paper, a non-cooperative Cournot Nash game is constructed to analyze the issue of marital instability and childbearing and two main results are obtained. First, we find that each spouse is less inclined to dissolve marriage with more kids in the family since the loss in the economies of scale associated with childcare is greater with a larger number of children. Second, contrary to the study done by Lillard and Waite, the model predicts that women do not always become less likely to bear children as the probability of divorce rises. Due to the utilities derived from children and the comparative advantage they have in earning labor income, childless wives who have higher wage rates than their husbands actually tend to give birth as a response to marital separation.

Using the data from the NLSY79, a system of childbearing and marital disruption equations is estimated to test the theoretical predictions from the model. The estimation results strongly support the notion that the addition of children to the family stabilizes marriage and weakly confirm the hypothesis that high-wage women (relative to the husbands) without prior kids are more likely to bear children as the probability of divorce increases.

Note: All the percentages in table 3.1-3.6 are the proportions of women who have given birth in the past two years

Table 3.1: Women who have prior kid (wife's wage > < husband's wage).

	Percentage	Standard Deviation	Number
Divorced women with higher wages than husbands	0.394	0.490	147
Non-divorced women with higher wages than husbands	0.503	0.502	147
Divorced women with lower wages than husbands	0.341	0.475	223
Non-divorced women with lower wages than husbands	0.462	0.499	470

Table 3.2: Women who do not have prior kids (wife's wage > < husband's wage).

	Percentage	Standard Deviation	Number
Divorced women with higher wages than husbands	0.161	0.368	267
Non-divorced women with higher wages than husbands	0.121	0.327	198
Divorced women with lower wages than husbands	0.136	0.343	331
Non-divorced women with lower wages than husbands	0.096	0.295	365

Table 3.3: Women who have prior kids (wife's wage > < 1.5 × husband's wage).

	Percentage	Standard Deviation	Number
Divorced women with wages > 1.5 × husbands' wages	0.468	0.502	94
Non-divorced women with wages > 1.5 × husbands' wages	0.523	0.502	88
Divorced women with wages < 1.5 × husbands' wages	0.326	0.470	276
Non-divorced women with wages < 1.5 × husbands' wages	0.463	0.499	529

Table 3.4: Women who do not have prior kids (wife's wage > < 1.5 × husband's wage).

	Percentage	Standard Deviation	Number
Divorced women with wages > 1.5 × husbands' wages	0.167	0.374	203
Non-divorced women with wages > 1.5 × husbands' wages	0.111	0.315	135
Divorced women with wages < 1.5 × husbands' wages	0.137	0.344	395
Non-divorced women with wages < 1.5 × husbands' wages	0.103	0.304	428

Table 3.5: Women who have prior kids (wife's wage > < 2 × husband's wage).

	Percentage	Standard Deviation	Number
Divorced women with wages > 2 × husbands' wages	0.507	0.503	75
Non-divorced women with wages > 2 × husbands' wages	0.557	0.500	70
Divorced women with wages < 2 × husbands' wages	0.325	0.469	295
Non-divorced women with wages < 2 × husbands' wages	0.461	0.499	547

Table 3.6: Women who do not have prior kids (wife's wage > < 2 × husband's wage).

	Percentage	Standard Deviation	Number
Divorced women with wages > 2 × husbands' wages	0.173	0.380	173
Non-divorced women with wages > 2 × husbands' wages	0.093	0.292	118
Divorced women with wages < 2 × husbands' wages	0.136	0.344	425
Non-divorced women with wages < 2 × husbands' wages	0.108	0.311	445

Table 3.7: Description of Variables

Variable	Description
Age92	Age of the wife in 1992
Sage92	Age of the husband in 1992
Education	Years of education for the wife
Owage89	Wage rate of the wife in 1989
Swage89	Wage rate of the husband in 1989
Difference	The difference between the wife's wage and the husband's wage in 1989 (= Owage89 - Swage89)
Dwage189	Dummy, = 1 if the wife's wage is greater than the husband's wage
Dwage289	Dummy, = 1 if the wife's wage is 1.5 times greater than the husband's wage
Dwage389	Dummy, = 1 if the wife's wage is 2 times greater than the husband's wage
Black	Dummy, = 1 if the wife is black
Siblings	Number of siblings for the wife
Catholic	Dummy, = 1 if the wife is Catholic
Jewish	Dummy, = 1 if the wife is Jewish
Relifreq	Frequency of religious attendance (scaled between 1-6 with 1 = never)
Nonurban	Dummy, = 1 if residence is in an nonurban area
South	Dummy, = 1 if residence is in the South
Before	Dummy, = 1 if the couple lived together before marriage
Second	Dummy, = 1 if the wife is in second marriage or above
Tenure	Years of marriage
$Tenure^2$	Tenure squared
Kidbm	Dummy, = 1 if there are kids from outside the current marriage
Number92	Number of children in the household in 1992
Noprekid	Dummy, = 1 if there are no kids in the household before 1990
Birth92	Dummy, = 1 if the wife gives birth between 1990 and 1992
Divorce	Dummy, = 1 if the couple has a divorce between 1993 and 1998

Table 3.8: Means and Standard Deviations of Variables

Variable	Mean (Standard Deviation)
Age92	30.85 (2.21)
Sage92	34.48 (4.62)
Education	12.78 (2.49)
Owage89	7.06 (5.86)
Swage89	11.18 (12.47)
Dwage189	0.32 (0.47)
Dwage289	0.17 (0.38)
Dwage389	0.13 (0.34)
Black	0.20 (0.40)
Siblings	3.89 (2.74)
Catholic	0.39 (0.49)
Jewish	0.01 (0.09)
Relifreq	3.25 (1.67)
Nonurban	0.22 (0.41)
South	0.40 (0.49)
Before	0.32 (0.47)
Second	0.09 (0.29)
Tenure	9.30 (3.58)
Kidbm	0.05 (0.21)
Number92	1.90 (1.19)
Noprekid	0.20 (0.40)
Birth92	0.31 (0.46)
Divorce	0.16 (0.37)

Number of Observations = 1869

Table 3.9: Reduced-Form Probit Equation for Divorce (regressed separately against husband's wage and wife's wage)

Variable	Parameter Estimate (Absolute Value of T-Statistic)
Constant	1.3786 (2.16)
Age92	-0.0803 (3.93)
Sage92	0.0087 (1.04)
Owage89	-0.0147 (1.94)
Swage89	-0.0081 (1.92)
Black	0.0139 (0.14)
Siblings	0.0060 (0.45)
Catholic	-0.1085 (1.31)
Relifreq	-0.0502 (2.21)
Nonurban	-0.0983 (1.08)
South	0.0834 (1.04)
Before	0.3275 (4.06)
Second	0.2537 (2.01)
Tenure	-0.0442 (0.86)
$Tenure^2$	0.0038 (1.53)
Noprekid	0.1737 (1.80)

Table 3.10: Structural Probit Equation for Divorce (regressed separately against husband's wage and wife's wage)

Variable	Parameter Estimate	Marginal Effect (dP/dX)
Constant	3.4793 (4.35)	0.8154
Age92	-0.0822 (4.03)	-0.0193
Sage92	-0.5088 (0.57)	-0.0012
Black	-0.1951 (1.65)	-0.0457
Catholic	-0.0649 (0.79)	-0.0152
Relifreq	-0.0307 (1.31)	-0.0072
Nonurban	-0.1592 (1.71)	-0.0373
South	-0.0595 (0.66)	-0.0139
Before	0.1953 (2.23)	0.0458
Second	0.0696 (0.52)	0.0163
Tenure	-0.1292 (2.44)	-0.0302
$Tenure^2$	0.0034 (1.38)	0.0008
Probability of Birth	-2.5244 (3.63)	-0.5916

Note:
1. Absolute values of t-statistics are shown in parentheses.

2. Probability of Birth is the predicted probability of childbirth obtained from the first stage.

Table 3.11: Reduced-Form Probit Equation for Childbirth From 1990-1992 (regressed separately against the log of husband's wage and the log of wife's wage)

Variable	Parameter Estimate (Absolute Value of T-Statistic)
Constant	0.5926 (1.03)
Age92	0.0047 (0.27)
Sage92	-0.0180 (2.20)
Owage89	0.0164 (3.00)
Swage89	0.0025 (0.97)
Black	-0.3267 (3.46)
Siblings	-0.0050 (0.39)
Catholic	0.0371 (0.51)
Relifreq	0.0260 (1.28)
Nonurban	-0.1040 (1.26)
South	-0.1904 (2.66)
Before	-0.1596 (2.16)
Second	-0.2460 (2.10)
Tenure	0.0166 (0.30)
$Tenure^2$	-0.0078 (2.65)
Noprekid	-0.2849 (3.28)

Table 3.12: Structural Probit Equation for Child-Birth (regressed separately against the log of husband's wage and the log of wife' s wage)

Variable	Parameter Estimate	Marginal Effect (dP/dX)
Constant	2.0132 (2.27)	0.6352
Age92	-0.0319 (1.31)	-0.0101
Sage92	-0.0139 (1.62)	-0.0044
Owage89	0.0064 (0.66)	0.0020
Swage89	-0.0016 (0.43)	-0.0005
Black	-0.3094 (3.25)	-0.0976
Siblings	-0.0021 (0.16)	-0.0007
Catholic	-0.0148 (0.20)	-0.0047
Relifreq	0.0048 (0.19)	0.0015
Nonurban	-0.1400 (1.63)	-0.0442
South	-0.1439 (1.97)	-0.0454
Second	-0.1232 (0.89)	-0.0389
Tenure	0.0028 (0.05)	0.0009
Tenure^2	-0.0064 (2.09)	-0.0020
Noprekid	-1.2451 (2.96)	-0.3929
Probability of Divorce	-2.4018 (2.27)	-0.7579
Inter1	0.0030 (0.03)	0.0009
Inter2	0.0296 (0.64)	0.0093
Inter3	0.0859 (2.29)	0.0271
Inter4	0.0156 (1.07)	0.0049
Inter5	5.2101 (2.60)	1.6440
Inter6	-0.3664 (1.64)	-0.1156
Inter7	-0.1699 (1.51)	-0.0536

Note:
1. Absolute values of t-statistics are shown in parentheses.
2. Probability of Divorce is the predicted probability of divorce from the first stage.
3. Inter1 = Owage89*Probability of Divorce;
4. Inter2 = Swage89*Probability of Divorce;
5. Inter3 = Owage89*Noprekid;
6. Inter4 = Swage89*Noprekid;
7. Inter5 = Noprekid*Probability of Divorce;
8. Inter6 = Owage89*Noprekid*Probability of Divorce;
9. Inter7 = Swage89*Noprekid*Probability of Divorce;

Table 3.13: Reduced-Form Probit Equation for Divorce (regressed against the difference between wife's wage and husband's wage)

Variable	Parameter Estimate (Absolute Value of T-Statistic)
Constant	1.4420 (2.26)
Age92	-0.0887 (4.39)
Sage92	0.0074 (0.89)
Difference	0.0035 (1.09)
Black	0.0411 (0.41)
Siblings	0.0113 (0.85)
Catholic	-0.1089 (1.32)
Relifreq	-0.0539 (2.38)
Nonurban	-0.0716 (0.80)
South	0.0950 (1.20)
Before	0.3283 (4.08)
Second	0.2803 (2.23)
Tenure	-0.0410 (0.79)
$Tenure^2$	0.0039 (1.59)
Noprekid	0.1523 (1.59)

Table 3.14: Structural Probit Equation for Divorce (regressed against the difference between wife's wage and husband's wage)

Variable	Parameter Estimate	Marginal Effect (dP/dX)
Constant	3.1837 (3.76)	0.7496
Age92	-0.0821 (4.02)	-0.0193
Sage92	-0.0034 (0.37)	-0.0008
Black	-0.1569 (1.25)	-0.0369
Catholic	-0.0702 (0.86)	-0.0165
Relifreq	-0.0337 (1.43)	-0.0079
Nonurban	-0.1477 (1.57)	-0.0348
South	-0.0356 (0.38)	-0.0084
Before	0.2122 (2.34)	0.0499
Second	0.0943 (0.69)	0.0222
Tenure	-0.1193 (2.21)	-0.0281
$Tenure^2$	0.0036 (1.43)	0.0008
Probability of Birth	-2.1516 (2.62)	-0.5066

Note:
1. Absolute values of t-statistics are shown in parentheses.
2. Probability of Birth is the predicted probability of childbirth obtained from the first stage.

Table 3.15: Reduced-Form Probit Equation for Childbirth (regressed against the difference between the wife's wage and the husband's wage)

Variable	Parameter Estimate (Absolute Value of T-Statistic)
Constant	0.4663 (0.81)
Age92	0.0133 (0.77)
Sage92	-0.0164 (2.03)
Difference	0.0006 (0.23)
Black	-0.3435 (3.65)
Siblings	-0.0096 (0.77)
Catholic	0.0402 (0.56)
Relifreq	0.0289 (1.42)
Nonurban	-0.1292 (1.57)
South	-0.1963 (2.74)
Before	-0.1616 (2.19)
Second	-0.2704 (2.31)
Tenure	0.0169 (0.30)
$Tenure^2$	-0.0081 (2.75)
Noprekid	-0.2636 (3.05)

Table 3.16: Second-Stage Probit Equation for Childbirth (regressed against the difference between wife's wage and husband's wage)

Variable	Parameter Estimate	Marginal Effect (dP/dX)
Constant	1.7467 (1.94)	0.5550
Age92	-0.0237 (0.94)	-0.0075
Sage92	-0.0125 (1.48)	-0.0040
Difference	0.0011 (0.21)	0.0004
Black	-0.3249 (3.43)	-0.1032
Siblings	-0.0043 (0.33)	-0.0014
Catholic	-0.0087 (0.12)	-0.0028
Relifreq	0.0075 (0.30)	0.0024
Nonurban	-0.1551 (1.85)	-0.0493
South	-0.1553 (2.12)	-0.0493
Second	-0.1411 (1.01)	-0.0448
Tenure	0.0026 (0.04)	0.0008
$Tenure^2$	-0.0066 (2.15)	-0.0021
Noprekid	-0.3252 (1.50)	-0.1033
Probability of Divorce	-1.94 (2.01)	-0.6150
Inter1	0.0001 (0.00)	0.0000
Inter2	-0.0223 (1.35)	-0.0071
Inter3	0.7929 (0.70)	0.2519
Inter4	0.1949 (1.85)	0.0619

Note:
1. Absolute values of t-statistics are shown in parentheses.
2. Probability of Birth is the predicted probability of childbirth obtained from the first stage.
3. Inter1 = Difference*Probability of Divorce;
4. Inter2 = Difference*Noprekid;
5. Inter3 = Noprekid*Probability of Divorce;
6. Inter4 = Difference*Noprekid*Probability of Divorce;

Chapter 4: Summary and Future Research Direction

In the conclusion of their paper, Lundberg and Pollak (1993) expressed doubts about the attempt to model marriage as a noncooperative alternating game. In particular, they used a quote from Martin Shubik (1989, p.103) which is stated as follows:

> The game in extensive form provides a process account of the detail of individual moves and information structure; the tree structure often employed in its description enables the researcher to keep track of the full history of any play of the game. This is useful for the analysis of reasonably well-structured formal process models where the beginning, end and sequencing of moves is well-defined, but is generally not so useful to describe complex, loosely structured social interaction.

Perhaps Shubik is right in that it is not generally suitable to specify a "complex, loosely structured social interaction" such as marriage as a noncooperative extensive game. Nonetheless, it can be useful to model certain aspects of marriage in a noncooperative framework if the nature of interaction is clearly stated and "the beginning, end and sequencing of moves" are carefully defined. In this thesis, we use a noncooperative two-person game to analyze the problem of moral hazard within marriage.

Theories regarding moral hazards associated with marriages maintain that individuals who specialize in household production (traditionally women) are inclined to lessen domestic obligations and raise labor market activities as probabilities of marital separation rise. This notion of self-insurance against the potential marital disruption also seems to be consistent with several empirical studies. The research in the past, however, did not examine the issue in an intertemporal optimization framework and as a result, failed to take into account crucial aspects of decision-making in failing marriages such as the value of marriage, specialization within households and the wellbeing after divorce. Once the individual optimal responses are linked to the value of marriage, division of labor, and the post-marital welfare, we derive a new set of predictions regarding the behaviors of women in disrupted marriages.

In Chapter 2, assuming the number of children in the family is fixed, we find that a woman would increase the level of labor supply as a response to marital instability only if her wage growth is greater than that of her husband. Conversely, if her husband's wage growth exceeds hers, she would reduce labor market activities. The reason that women would actually work less as a response to the rising likelihood of separation is as follows. Women with low wage growth (relative to the husbands) can contribute more to the spouses' welfare by spending more time on the production of domestic goods. If they attempt to improve the earning power in the state of divorce by increasing the amount of time devoted to the labor market, they diminish the gains from marriage for the husbands and as a consequence, the husbands are more likely to leave them. The time-allocation decision faced by these women at the margin is essentially making the tradeoff between the utility attainable as a single person and the longevity of the marriage. Since the benefits from a longer marriage are likely to outweigh the additional wage gains from working in the market for wives with low wage growth, these women would work less as marriage becomes less stable.

Using the data from the NLSY79, we test the validity of the theoretical prediction regarding labor supply, divorce probability and difference in wage growth. In examining the behavior related to labor-force participation, we find evidence supports the hypothesis that women with lower wage growth than their husbands are likely to reduce labor supply as the probability of divorce rises. However, the results also suggest that women with higher relative wage growth would be averse to join the labor force as well, although to a much lesser extent. When analyzing the number of work hours, the results show that the expectation of marital separation leads to more hours worked and the magnitudes of the increase are roughly the same for women with different relative wage growth.

In Chapter 3, we relax the assumption of the constant number of kids in the household and thereby incorporate fertility decisions in the model. Contrary to the previous studies, the analysis shows that women do not always become less likely to give birth as the likelihood of divorce rises. Wives who do not have any kids and who have higher wage rates than their husbands actually possess the tendency to bear children as their marriages disintegrate. The rationale behind this seemingly odd fertility response to marital instability is the following. A woman who does not have prior children may like to have a child as the marriage is ending[35]; however, she is likely to handle the majority of the childcare if her wage rate is lower than that of her husband due to the optimal rule regarding division of labor. The intensified effort made by the wife in the household production translates into a lower potential wage rate and consequently, a reduction in the welfare in the state of divorce. If the woman has a high wage rate relative to her

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[35] Recall that in the model, a woman is not allowed to remarry and have additional kids once divorced. Hence, a woman's desire to have children should be particularly strong at the end of the marriage.

husband, on the other hand, the specialization of tasks will lead the man to share a large portion of the responsibilities related to raising the child. Hence, only childless women who have wages greater than those of the husbands are inclined to bear children at the brink of marital dissolution.

Again, we use the data from the NLSY79 to test the hypothesis regarding fertility. The estimation results are consistent with the theoretical prediction. We find that a 10% increase in the probability of divorce, coupled with a $5 increase in the wage difference between the wife and husband would raise the probability of bearing children by 3% for childless women.

The model presented in this thesis revealed how individuals would respond to adversities in marriage. The natural extension of the theory is to endogenize the mating choice in the initial period. That is, instead of simply assuming the couple would agree to marry upon observing the compatibility parameter, we allow the two individuals to negotiate over the terms of the marriage contract. In the case that an agreement cannot be reached, the union between the two people does not take place. The introduction of prenuptial negotiations will enable us to explore how marriage contracts are constructed in anticipation to possible strategic behaviors that arise in unstable relationships. Once we understand the structure of marriage contracts in relation to optimal responses within marriage, the next step is to investigate the constraints imposed on the contracts by no-fault divorce laws and in turn identify the effects of no-fault divorce on labor supply, fertility, and the likelihood of marital separation.

In an illuminating paper that shed light on the link between information constraints and marital contracting, Peters (1986) analyzed the impact of no-fault divorce on various aspects of marriage. In particular, two models with different assumptions regarding the information about options outside marriage are compared. In the model where both spouses' outside options are public knowledge, the law does not change the likelihood of separation[36] but alters the compensation scheme at divorce[37]. In the model where each spouse only has information about her/his own outside options but not about the alternatives faced by the other, no-fault divorce raises the probabilities of marital dissolution but has no effect on the compensation scheme[38]. Utilizing data from the Current Population Survey (March/April 1979), Peters tested the validity of the two models and found strong evidence in support of the perfect information hypothesis. The empirical results showed that the divorce rates are not statistically different in unilateral and mutual consent states and that compensating payments are less volatile in unilateral states.

The theoretical predictions from Peters' models, however, are based on the assumption of fixed investments in marriage-specific capital. Realizing that the assumption of invariant marriage-specific investments is not always valid, Peters talked about the issue of "moral hazard" as an addendum to her main framework. Nonetheless, the discussion was under the premise that women would always reduce the amount of

[36] In a world with perfect information, the agents can freely bargain over the gains to marriage and divorce only occurs when the total utilities in the divorced state exceed the total utilities in the married state, regardless the rule of separation.

[37] Under unilateral divorce, the person who is better off in the state of divorce is not required to compensate the other party. On the other hand, under mutual divorce, one needs to compensate the spouse by an amount that exceeds the reduction in welfare from being divorced, otherwise the separation would not occur. The required compensating payment varies from one case to the next and hence one would expect the variance of divorce compensation to be greater in mutual states.

[38] In the absence of perfect information, the optimal marriage contract calls for a fixed wage payment from one party to the other in order to eliminate costly *ex post* bargaining. The negative consequence of the fixed transfer, however, is the more-than-efficient number of separations.

time spent on household production in response to the possibility of divorce. According to Peters, as long as the problem of moral hazard can be anticipated, there will be provisions established in the initial contract to alleviate the undesirable consequences. One scheme that can be used to compensate the wife for the time spent on housework is the specification of divorce settlement payments. Unilateral divorce laws, however, may cause difficulties in enforcing the compensation payments and thereby raise the costs of implementing the terms of the contract. It follows that, in equilibrium, there is likely to be a smaller number of marriage contracts negotiated under the unilateral rule.

The notion that women would always respond to marital instability by decreasing the level of domestic activities is shown by this thesis to be false. Consequently, the analysis based on that assumption is also likely to be incorrect. We outline a two-stage game below to illustrate how the framework adopted in this thesis can be extended to examine the structure of marriage contract and the impact of no-fault divorce.

At the beginning of the first period, two individuals meet in the marriage market and observe a set of personal characteristics, including compatibility between them. If they deem each other to be acceptable mates, the negotiation for a marriage contract ensues.[39] For the sake of simplicity, suppose the contract only specifies the amount of transfers from one spouse to the other if separation occurs in the second period. If bargaining over the appropriate level of transfers breaks down, each person proceeds to search for a new potential partner and the game ends. On the other hand, if an agreement can be reached, both spouses make labor supply and fertility choices in the same fashion as described in the thesis. At the beginning of the second period, a new compatibility parameter is observed and the decision about whether or not to dissolve marriage is made. If divorce occurs, a pre-specified amount of money changes hands between the two.

Although the transfer is specified by the contract, it can take on different forms. One option is to have a constant level of transfer C. Another possibility is to make the transfer contingent upon the levels of investment in household production; i.e., C = C(R) where R is the amount of time spent on domestic activities. Obviously, the optimal rule for transfers varies under different scenarios. But, in any case, the determination of optimal transfers should be a function of the expected actions in the first period, and, as the thesis has demonstrated, actions within marriage depend on the anticipated compatibility in the future. Hence, there is a critical link between the structure of the marriage contract, responses within marriage, and marital instability. Since unilateral divorce laws may alter the arrangement of the contracts, these rules are also likely to impact behaviors regarding labor supply, childbearing, and marital disruption.

Finally, the model adopted in this thesis can be applied to any bilateral monopoly situation with slight modifications. For instance, the association between a firm and its employee can be characterized as a union where there are mutual benefits as well as possibilities of separation (in the forms of quits and layoffs). The stochastic element that causes the dissolution of the labor-firm relationship can be fluctuation in demand or advance in technology. Using our general framework, we can study the effects of

[39] We are not assuming that these two people are the best possible match in the market. However, if there were nontrivial search costs, they would settle for someone who is sufficiently desirable.

changes in consumer demand and costs of production on the optimal responses by workers and firms.

Appendix 2.1:

From equation (2.17), we know that:

(A2.1) $$q_t^i * = \ln w_t^i + \ln L_t^i(d) + \ln g_t^i + \ln(H - L_t^i(d)) + bV_{t+1}^i(d)$$

$$- a^i \ln w_t^i - a^i \ln L_t^i(m) - b^i \ln w_t^j - b^i \ln L_t^j(m)$$

$$- \ln[g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))] - bV_{t+1}^i(q_{t+1} | q_t^i *)$$

Differentiating (A2.1) with respect to L^i_t, we have:

(A2.2) $$\frac{\partial q_t^i *}{\partial L_t^i} = \frac{w^{i\,'}}{w_{t+1}^i} - \frac{a^i w^{i\,'}}{w_{t+1}^i} = (1 - a^i)\frac{w^{i\,'}}{w_{t+1}^i} > 0$$

Differentiating (A2.2) with respect to L^i_t, we have:

(A2.3) $$\frac{\partial^2 q_t^i *}{\partial L_t^{i\,2}} = -(1 - a^i)(\frac{w^{i\,'}}{w_{t+1}^i})^2 + (1 - a^i)\frac{w^{i\,''}}{w_{t+1}^i} < 0$$

Differentiating (A2.2) with respect to L^j_t, we have:

(A2.4) $$\frac{\partial^2 q_t^i *}{\partial L_t^i \partial L_t^j} = 0$$

Differentiating (A2.1) with respect to L^j_t, we have:

(A2.5) $$\frac{\partial q_t^i *}{\partial L_t^j} = -\frac{b^i w^{j\,'}}{w_{t+1}^j} < 0$$

Differentiating (A2.5) with respect to L^j_t, we have:

(A2.6) $$\frac{\partial^2 q_t^i *}{\partial L_t^{j\,2}} = b^i (\frac{w^{j\,\prime}}{w_{t+1}^j})^2 - b^i \frac{w^{j\,\prime\prime}}{w_{t+1}^j} > 0$$

Finally, differentiating (A2.5) with respect to L^j_t, we have:

(A2.7) $$\frac{\partial^2 q_t^i *}{\partial L_t^j \partial L_t^i} = 0$$

Appendix 2.2:

It is unclear what the sign of $\begin{vmatrix} \frac{\P^{\,2} V_t^i}{\P L_t^{i\,2}} & \frac{\P^{\,2} V_t^i}{\P L_t^i \P L_t^j} \\ \frac{\P^{\,2} V_t^j}{\P L_t^j \P L_t^i} & \frac{\P^{\,2} V_t^j}{\P L_t^{j\,2}} \end{vmatrix}$ would be without making simplifying assumptions regarding the wage growth of the household members (?$^{i\prime}$ and ?$^{j\prime}$). If we assume sufficiently flat wage profiles for both person i and j, i.e., both ?$^{i\prime}$ and ?$^{j\prime}$ are close to zero. Then we have:

(A2.8) $$\frac{\partial^2 V_t^i}{\partial L_t^{i\,2}} \frac{\partial^2 V_t^j}{\partial L_t^{j\,2}}$$

$$= \{-\frac{a^i}{L_t^{i\,2}} - \frac{g_t^{i\,2}}{[g_t^i(H-L_t^i)+g_t^j(H-L_t^j)]^2}\}\{-\frac{a^j}{L_t^{j\,2}} - \frac{g_t^{j\,2}}{[g_t^i(H-L_t^i)+g_t^j(H-L_t^j)]^2}\}$$

$$= \frac{a^i a^j}{L_t^{i\,2} L_t^{j\,2}} + \frac{a^i g_t^{j\,2}}{[g_t^i(H-L_t^i)+g_t^j(H-L_t^j)]^2 L_t^{i\,2}} +$$

$$\frac{a^i g_t^{i\,2}}{[g_t^i(H-L_t^i)+g_t^j(H-L_t^j)]^2 L_t^{j\,2}} + \frac{g_t^{i\,2} g_t^{j\,2}}{[g_t^i(H-L_t^i)+g_t^j(H-L_t^j)]^4}$$

(A2.9) $$\frac{\partial^2 V_t^i}{\partial L_t^i \partial L_t^j} \frac{\partial^2 V_t^j}{\partial L_t^j \partial L_t^i}$$

$$= \{-\frac{g_t^i g_t^j}{[g_t^i(H-L_t^i)+g_t^j(H-L_t^j)]^2}\}\{-\frac{g_t^i g_t^j}{[g_t^i(H-L_t^i)+g_t^j(H-L_t^j)]^2}\}$$

$$= \frac{g_t^{i\,2} g_t^{j\,2}}{[g_t^i (H - L_t^i) + g_t^j (H - L_t^j)]^4}$$

(A2.10) $$\begin{vmatrix} \dfrac{\P^2 V_t^i}{\P L_t^{i\,2}} & \dfrac{\P^2 V_t^i}{\P L_t^i \P L_t^j} \\ \dfrac{\P^2 V_t^j}{\P L_t^j \P L_t^i} & \dfrac{\P^2 V_t^j}{\P L_t^{j\,2}} \end{vmatrix}$$

$$= \frac{a^i a^j}{L_t^{i\,2} L_t^{j\,2}} + \frac{a^i g_t^{j\,2}}{[g_t^i (H - L_t^i) + g_t^j (H - L_t^j)]^2 L_t^{i\,2}} + \frac{a^i g_t^{i\,2}}{[g_t^i (H - L_t^i) + g_t^j (H - L_t^j)]^2 L_t^{j\,2}}$$

> 0

Hence, the sufficient condition for local stability of the dynamical system is satisfied.

Appendix 3.1:

From equation (3.11), we can express the marginal effect of new children chosen by person i on the probability of staying married (from person i's perspective) as:

$$(A3.1)\qquad \frac{\partial P_t^i}{\partial n_{t-1}^i} = \Big[\frac{g_t^i(H-L_t^i(d))}{g_t^i(H-L_t^i(d))-N_{t-1}\bar{Q}} - \frac{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))}{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))-N_{t-1}\bar{Q}}\Big]\frac{\partial N_{t-1}}{\partial n_{t-1}^i}$$

From equation (3.12), the marginal effect of new children chosen by person j on the probability of staying married (from person i's perspective) is:

$$(A3.2)\qquad \frac{\partial P_t^i}{\partial n_{t-1}^j} = \Big[\frac{g_t^i(H-L_t^i(d))}{g_t^i(H-L_t^i(d))-N_{t-1}\bar{Q}} - \frac{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))}{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))-N_{t-1}\bar{Q}}\Big]\frac{\partial N_{t-1}}{\partial n_{t-1}^j}$$

It follows from (A3.1) and (A3.2) that:

$$(A3.3)\qquad \frac{\partial^2 P_t^i}{\partial {n_{t-1}^i}^2} = \Big\{\frac{g_t^i(H-L_t^i(d))\bar{Q}}{[g_t^i(H-L_t^i(d))-N_{t-1}\bar{Q}]^2} - \frac{[g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))]\bar{Q}}{\{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))-N_{t-1}\bar{Q}\}^2}\Big](\frac{\partial N_{t-1}}{\partial n_{t-1}^i})^2$$

> 0

(A3.4) $$\frac{\partial^2 P_t^i}{\partial n_{t-1}^i \partial n_{t-1}^j} = \{\frac{g_t^i(H - L_t^i(d))\bar{Q}}{[g_t^i(H - L_t^i(d)) - N_{t-1}\bar{Q}]^2}$$

$$- \frac{[g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))]\bar{Q}}{\{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m)) - N_{t-1}\bar{Q}\}^2}]\frac{\partial N_{t-1}}{\partial n_{t-1}^i}\frac{\partial N_{t-1}}{\partial n_{t-1}^j}$$

> 0

(A3.5) $$\frac{\partial^2 P_t^i}{\partial n_{t-1}^{j\ 2}} = \{\frac{g_t^i(H - L_t^i(d))\bar{Q}}{[g_t^i(H - L_t^i(d)) - N_{t-1}\bar{Q}]^2}$$

$$- \frac{[g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))]\bar{Q}}{\{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m)) - N_{t-1}\bar{Q}\}^2}](\frac{\partial N_{t-1}}{\partial n_{t-1}^j})^2$$

> 0

Appendix 3.2:

Part 1: D1 = $\frac{\partial^2 P_t^j}{\partial {n_t^j}^2}[V_{t+1}^j(m) - V_{t+1}^j(d)] - \frac{\partial^2 P_t^i}{\partial n_t^i \partial n_t^j}[V_{t+1}^i(m) - V_{t+1}^i(d)]$

From Appendix 3.1, we know that:

(A3.6)
$$\frac{\partial^2 P_t^j}{\partial {n_{t-1}^j}^2} = \{\frac{g_t^j(H - L_t^j(d))\bar{Q}}{[g_t^j(H - L_t^j(d)) - N_{t-1}\bar{Q}]^2} - \frac{[g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))]\bar{Q}}{\{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m)) - N_{t-1}\bar{Q}\}^2}](\frac{\partial N_{t-1}}{\partial n_{t-1}^j})^2$$

(A3.7)
$$\frac{\partial^2 P_t^i}{\partial n_{t-1}^i \partial n_{t-1}^j} = \{\frac{g_t^i(H - L_t^i(d))\bar{Q}}{[g_t^i(H - L_t^i(d)) - N_{t-1}\bar{Q}]^2} - \frac{[g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m))]\bar{Q}}{\{g_t^i(H - L_t^i(m)) + g_t^j(H - L_t^j(m)) - N_{t-1}\bar{Q}\}^2}]\frac{\partial N_{t-1}}{\partial n_{t-1}^i}\frac{\partial N_{t-1}}{\partial n_{t-1}^j}$$

Comparing (A3.6) and (A3.7), $\frac{\partial^2 P_t^j}{\partial {n_{t-1}^j}^2}$ is greater than $\frac{\partial^2 P_t^i}{\partial n_{t-1}^i \partial n_{t-1}^j}$ if $L_t^j(d)$ is greater than $L_t^i(d)$. In Appendix 3.3, it is shown that an individual would work more in the state of divorce if he/she has a lower wage rate. Hence, $\frac{\partial^2 P_t^j}{\partial {n_{t-1}^j}^2}$ > $\frac{\partial^2 P_t^i}{\partial n_{t-1}^i \partial n_{t-1}^j}$ if person i has a higher wage rate than person j.

Also, $V_{t+1}^{j}(m)-V_{t+1}^{j}(d) > V_{t+1}^{i}(m)-V_{t+1}^{i}(d)$ if person i has a higher wage rate than person j. The reasoning is as follows. If $V_{t+1}^{j}(m)-V_{t+1}^{j}(d)$ is greater $V_{t+1}^{i}(m)-V_{t+1}^{i}(d)$, it means that person j benefits proportionately more from the marriage relative to person i. In the model, person i and person j are assumed to be equally efficient at housework (i.e., raising children), however, they may have different wage rates. Because of the semi-public nature of the consumption good and the pure-public property of children within marriage, if person j derives more utilities from the marriage than person i does, person j must have a lower wage rate than person i.

Therefore, we can conclude that as long as person i has a higher wage rate than person j, D1 > 0.

Part 2: D2 = $\dfrac{\partial P_t^j}{\partial n_t^j}[\dfrac{\partial V_{t+1}^{j}(m)}{\partial n_t^j}-\dfrac{\partial V_{t+1}^{j}(d)}{\partial n_t^j}]-\dfrac{\partial P_t^i}{\partial n_t^i}[\dfrac{\partial V_{t+1}^{i}(m)}{\partial n_t^j}-\dfrac{\partial V_{t+1}^{i}(d)}{\partial n_t^j}]$

From Appendix 3.1, we have:

(A3.8) $$\frac{\partial P_t^j}{\partial n_{t-1}^j} = [\frac{g_t^i(H-L_t^j(d))}{g_t^i(H-L_t^j(d))-N_{t-1}\bar{Q}} - \frac{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))}{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))-N_{t-1}\bar{Q}}]\frac{\partial N_{t-1}}{\partial n_{t-1}^j}$$

(A3.9) $$\frac{\partial P_t^i}{\partial n_{t-1}^i} = [\frac{g_t^i(H-L_t^i(d))}{g_t^i(H-L_t^i(d))-N_{t-1}\bar{Q}}$$

$$-\frac{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))}{g_t^i(H-L_t^i(m))+g_t^j(H-L_t^j(m))-N_{t-1}\bar{Q}}]\frac{\partial N_{t-1}}{\partial n_{t-1}^i}$$

As we can see from (A3.8) and (A3.9), $\frac{\partial P_t^j}{\partial n_{t-1}^j} > \frac{\partial P_t^i}{\partial n_{t-1}^i}$ if $L_t^j(d) > L_t^i(d)$. Again, this implies that person i has a higher wage rate than person j.

The higher wage for person i relative to person j also means $\frac{\partial V_{t+1}^j(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^j(d)}{\partial n_t^j} > \frac{\partial V_{t+1}^i(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^i(d)}{\partial n_t^j}$. The value for each of these terms is written below:

(A3.10) $$\frac{\partial V_{t+1}^j(m)}{\partial n_t^j} = \frac{1}{N_t}[1-\frac{g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))}{g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))-N_t\bar{Q}}]\frac{\partial N_t}{\partial n_t^j}$$

(A3.11) $$\frac{\partial V_{t+1}^j(d)}{\partial n_t^j} = \frac{1}{N_t}[1-\frac{g_{t+1}^j(H-L_{t+1}^j(d))}{g_{t+1}^j(H-L_{t+1}^j(d))-N_t\bar{Q}}]\frac{\partial N_t}{\partial n_t^j}$$

(A3.12) $$\frac{\partial V_{t+1}^i(m)}{\partial n_t^j} = \frac{1}{N_t}[1-\frac{g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))}{g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))-N_t\bar{Q}}]\frac{\partial N_t}{\partial n_t^j}$$

(A3.13) $$\frac{\partial V_{t+1}^{i}(d)}{\partial n_t^j} = \frac{1}{N_t}[1 - \frac{g_{\ t+1}^{\,j}(H - L_{t+1}^{i}(d))}{g_{\ t+1}^{\,j}(H - L_{t+1}^{i}(d)) - N_t \bar{Q}}]\frac{\partial N_t}{\partial n_t^j}$$

Since $\frac{\partial V_{t+1}^{j}(m)}{\partial n_t^j} = \frac{\partial V_{t+1}^{i}(m)}{\partial n_t^j}$ and $\frac{\partial V_{t+1}^{j}(d)}{\partial n_t^j} < \frac{\partial V_{t+1}^{i}(d)}{\partial n_t^j}$ if $L_{t+1}^{j}(d) > L_{t+1}^{i}(d)$,

we can deduce that $\frac{\partial V_{t+1}^{j}(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^{j}(d)}{\partial n_t^j} > \frac{\partial V_{t+1}^{i}(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^{i}(d)}{\partial n_t^j}$ when person i has a higher wage rate than person j.

Hence, D2 > 0 if person i has a high wage relative to person j.

Part 3: D3 = $\frac{\partial P_t^j}{\partial n_t^j}[\frac{\partial V_{t+1}^{j}(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^{j}(d)}{\partial n_t^j}] - \frac{\partial P_t^i}{\partial n_t^j}[\frac{\partial V_{t+1}^{i}(m)}{\partial n_t^i} - \frac{\partial V_{t+1}^{i}(d)}{\partial n_t^i}]$

From Appendix 3.1, we see that $\frac{\partial P_t^j}{\partial n_t^j} > \frac{\partial P_t^i}{\partial n_t^j}$ if $L_{t+1}^{j}(d) > L_{t+1}^{i}(d)$. Also, $\frac{\partial V_{t+1}^{j}(m)}{\partial n_t^j} - \frac{\partial V_{t+1}^{j}(d)}{\partial n_t^j} > \frac{\partial V_{t+1}^{i}(m)}{\partial n_t^i} - \frac{\partial V_{t+1}^{i}(d)}{\partial n_t^i}$ if $L_{t+1}^{j}(d) > L_{t+1}^{i}(d)$. Thus, we can conclude that D3 > 0 in the case when person i has a higher wage rate than person j.

Part 4: D4 = $P_t^j \frac{\partial^2 V_{t+1}^{j}(m)}{\partial {n_t^j}^2} - P_t^i \frac{\partial^2 V_{t+1}^{i}(m)}{\partial n_t^i \partial n_t^j}$

The values for $\frac{\partial^2 V_{t+1}^{j}(m)}{\partial {n_t^j}^2}$ and $\frac{\partial^2 V_{t+1}^{i}(m)}{\partial n_t^i \partial n_t^j}$ are listed below:

(A3.14) $$\frac{\partial^2 V_{t+1}^j(m)}{\partial {n_t^j}^2} =$$

$$\{-\frac{1}{{N_t}^2}+\frac{g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))}{\{[g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))]N_t-\bar{Q}\,{N_t}^2\}^2}$$

$$\frac{[g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))]-2\bar{Q}\,N_t}{\{[g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))]N_t-\bar{Q}\,{N_t}^2\}^2}\}(\frac{\partial N_t}{\partial n_t^j})^2$$

(A3.15) $$\frac{\partial^2 V_{t+1}^i(m)}{\partial n_t^i \partial n_t^j} =$$

$$\{-\frac{1}{{N_t}^2}+\frac{g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))}{\{[g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))]N_t-\bar{Q}\,{N_t}^2\}^2}$$

$$\frac{[g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))]-2\bar{Q}\,N_t}{\{[g_{t+1}^i(H-L_{t+1}^i(m))+g_{t+1}^j(H-L_{t+1}^j(m))]N_t-\bar{Q}\,{N_t}^2\}^2}\}\frac{\partial N_t}{\partial n_t^j}\frac{\partial N_t}{\partial n_t^i}$$

From (A3.14) and (A3.15), we can infer that $\frac{\partial^2 V_{t+1}^j(m)}{\partial {n_t^j}^2} = \frac{\partial^2 V_{t+1}^i(m)}{\partial n_t^i \partial n_t^j}$. Hence, whether D4 is a positive or negative number depends on the relative size of P_t^j and P_t^i. D4 is positive if P_t^j is greater than P_t^i. $P_t^j > P_t^i$ means that person j is less willing to dissolve the marriage than person i. In our model, the lower potential wage rate for person j is responsible for a greater level of utilities derived by person j from the marriage. Therefore, the high wage rate for person i translates into $P_t^j > P_t^i$, which in turn leads to D4 > 0.

Part 5: D5 = $(1-P_t^j)\frac{\partial^2 V_{t+1}^j(d)}{\partial n_t^{j\,2}} - (1-P_t^i)\frac{\partial^2 V_{t+1}^i(d)}{\partial n_t^i \partial n_t^j}$

We can alternatively write D5 as:

(A3.16) D5 = $[\frac{\partial^2 V_{t+1}^j(d)}{\partial n_t^{j\,2}} - \frac{\partial^2 V_{t+1}^i(d)}{\partial n_t^i \partial n_t^j}] - [P_t^j \frac{\partial^2 V_{t+1}^j(d)}{\partial n_t^{j\,2}} - P_t^i \frac{\partial^2 V_{t+1}^i(d)}{\partial n_t^i \partial n_t^j}]$

The values for $\frac{\partial^2 V_{t+1}^j(d)}{\partial n_t^{j\,2}}$ and $\frac{\partial^2 V_{t+1}^i(d)}{\partial n_t^i \partial n_t^j}$ are listed below:

(A3.17) $\frac{\partial^2 V_{t+1}^j(d)}{\partial n_t^{j\,2}} =$

$$\{-\frac{1}{N_t^{\,2}} + \frac{[g_{\ t+1}^{\,i}(H-L_{t+1}^j(d))][g_{\ t+1}^{\,j}(H-L_{t+1}^j(d)) - 2\bar{Q}\,N_t]}{[g_{\ t+1}^{\,j}(H-L_{t+1}^j(d))]N_t - \bar{Q}\,N_t^{\,2}\}^2}\}(\frac{\partial N_t}{\partial n_t^j})^2$$

(A3.18) $\frac{\partial^2 V_{t+1}^i(d)}{\partial n_t^i \partial n_t^j} =$

$$\{-\frac{1}{N_t^{\,2}} + \frac{[g_{\ t+1}^{\,i}(H-L_{t+1}^i(d))][g_{\ t+1}^{\,i}(H-L_{t+1}^i(d)) - 2\bar{Q}\,N_t]}{[g_{\ t+1}^{\,i}(H-L_{t+1}^i(d))]N_t - \bar{Q}\,N_t^{\,2}\}^2}\}(\frac{\partial N_t}{\partial n_t^i})(\frac{\partial N_t}{\partial n_t^j})$$

When $L^{j}_{t+1}(d) > L^{i}_{t+1}(d)$ which also signifies a higher wage rate for person i than person j, $\dfrac{\partial^2 V^{j}_{t+1}(d)}{\partial n^{j\,2}_{t}}$ is greater than $\dfrac{\partial^2 V^{i}_{t+1}(d)}{\partial n^{i}_{t} \partial n^{j}_{t}}$. Also, a higher wage rate for person i implies that $P^{j}_{t} > P^{i}_{t}$. Hence, D5 > 0 if person i has a higher wage than person j.

Appendix 3.3:

Person i's value function in the state of divorce in period t is :

(A3.19) $$V_t^i(d) = \max_{L_t^i} \ln w_t^i + \ln L_t^i + \ln N_{t-1} + \ln[\frac{g_t^i(H - L_t^i)}{N_{t-1}} - \bar{Q}] + bV_{t+1}^i(d \mid L_t^i)$$

Differentiate (A19) with respect to L_t^i , we have the following first-order condition:

(A3.20) $$\frac{\partial V_t^i(d)}{\partial L_t^i} = \frac{1}{L_t^i} + \frac{1}{(1/g_t^i)\bar{Q}\,N_{t-1} - (H - L_t^i)} + \frac{w'(L_t^i)}{w_{t+1}^i} = 0$$

We perform the following comparative statics, $\frac{\partial L_t^i}{\partial w_0^i}$, i.e., how the labor supply changes with respect to the "endowed" wage rate. Since we can write (A3.20) as an identity, the sign of $\frac{\partial L_t^i}{\partial w_0}$ is the same as the sign of $\frac{\partial^2 V_t^i(d)}{\partial L_t^i \partial w_0^i}$ and $\frac{\partial^2 V_t^i(d)}{\partial L_t^i \partial w_0^i}$ is equal to:

(A3.21) $$\frac{\partial^2 V_t^i(d)}{\partial L_t^i \partial w_0^i} = -\frac{w'}{{w_{t+1}^i}^2} < 0$$

From (A3.21), we know that $\frac{\partial L_t^i}{\partial w^i} < 0$, hence an individual would work more in the state of divorce the lower the wage rate and would work less the higher the wage rate. The intuition behind this result is that quality of children, also an argument in the utility function, depends on the amount of time spent on the kids and someone with a high wage rate can "afford" to work less and devote more time to the children.

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Vita

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